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Dong Ngac Ward, Hanoi, Vietnam**CHARACTERIZATION OF MODE LOCALIZATION OF MICRO-SIZED TWO-SEGMENT TIMOSHENKO BEAMS
WITH DIFFERENT MATERIALS USING THE FINITE ELEMENT METHOD****Abstract**

Mode localization is a phenomenon observed in dynamic systems with weakly coupled elements, in which a small change in system parameters leads to a considerable concentration of vibration amplitude or vibration energy within a limited region instead of being distributed across the entire system. In this study, we focus on analyzing the mode localization characteristics of a two-segment beam structure made of different materials using the finite element method associated with energy criteria to evaluate the degree of mode localization. Timoshenko's beam theory is used in establishing the finite element equations. The advantage of using Timoshenko beams is that it can capture the shear effect in case of our beam because one of the two beam segments has a short length, while the cross-section remains constant, resulting in a relatively small slenderness ratio for the shorter segment, which is not suitable when using Euler-Bernoulli beam theory. In the framework of the free vibration problem, the proposed kinetic energy indicator is based on the ratio of kinetic energy between two beam segments, to assess the degree of mode localization along the beam length. The kinetic energy ratio depending on the beam's parameters is explored in detail.

Keywords:

mode localization, coupled systems, Timoshenko beam, finite element, energy ratio.

1. Introduction

Mode localization is a dynamic phenomenon arising from systems with weakly coupled components, where the amplitude or energy of vibration is concentrated in a particular region of the system [1, 2]. The concept of mode localization was first introduced by Anderson [3] when studying the localization characteristics of electrons in a disordered lattice. Throughout the 20th and early 21st centuries, Anderson's principle has been applied to explain many physical phenomena in various fields, related to the localization of energy in a region rather than its uniform distribution across all regions of a system with coupled elements, such as in acoustics [4], optics [5], chemistry [6], aerospace engineering [7].

In the field of structural engineering, particularly large-scale structures, a characteristic of mode localization is the confinement of energy within one or more structural elements, such as within a particular segment of a multi-span beam [8]. In [9], Bouzit and Piere examined the dynamic irregularities of multi-span beams to study the mode localization effect. They observed that if there exists a difference between the lengths of the beam spans, this leads to the dynamic response of the beam being localized within a small region along the beam's length. Another study by Pierre et al. [10] presented the vibration localization properties in a multi-span beam system, both theoretically and experimentally. The results showed that localization occurs when there is a small deviation in the lengths of the beam spans. These authors also proposed a criterion for predicting the appearance of strongly localized modes. Another aspect of localized systems can be found in a notable study by Hodges and Woodhouse [11]. They found in pendulum systems that even a small deviation in parameters within a periodic vibration model can lead to energy localization within the system.

In the field of micro- and nano-scale structures, recent progress in the fabrication technology of modern sensors has made, in part, thanks to new methods that increase the sensitivity of sensors compared to traditional methods based on frequency shift [12]. These sensors typically utilize beam-like structures or combinations of beams to detect some changes acting on the system structure. Unlike traditional single-degree-of-freedom resonant sensors that rely primarily on the principle of frequency shift during measurement, sensors based on the modal localization principle significantly increase the sensitivity of the system. This sensitivity is monitored through the change rate of modal shapes of a vibrating system in which a small change in a particular parameter causes a breaking of the initial symmetry of the system. Consequently, the energy concentration in certain modes of the system occurs. One of the effective applications of mode localization was discussed in the study by Spletzer et al. [13] for MEMS structures. That structure is a system of two cantilever beams connected to each other. At the end of one beam has a small mass, i.e., a disturbance in the beam's mass compared to the two original identical beams. The occurrence of the disturbance significantly affects the shape of the vibration mode and therefore allows for efficient detection of mass changes. The phenomenon of mode localization, arising from mode interaction, offers a potential solution to overcome the limitations of traditional sensor technology, as seen in the research of Wang et al. [14]. They present a decoupled-decomposition noise analysis model, which is applied to achieve high resolution in a mode-localized tilt sensor based on a coupled resonator closed-loop circuit. Zang et al. [15] proposed virtually coupled resonators with modal dominance for improved sensitivity and bandwidth. Their experimental results validate the feasibility of virtually coupled resonators and the double-closed-loop control. A notable result demonstrates a 2.7-fold improvement in sensitivity and at least a four-fold enhancement in bandwidth relative to weakly coupled resonators with identical parameters.

The MEMS structures discussed above are largely based on the Euler-Bernoulli beam theory. One drawback of the Euler-Bernoulli beam theory is that if the beam structure of the MEMS system becomes shorter and the height of beam's cross section becomes thicker, the shear effect needs to be taken into account [16]. Thus, in this study, the authors use the Timoshenko beam theory to investigate the mode localization effect of a micro-sized beam composed of two segments, each made from a different material. In MEMS systems using mode localization, a small change in system parameters can lead to a certain change in the localization properties. A feature of traditional MEMS structures is the use of at least two coupled beams, and their link may be a weak one. Our study proposes that even using only one beam with two different materials, the mode localization phenomenon is observed, i.e., there is a nonuniform energy distribution on the two segments of the beam. The mode localization properties are investigated using the finite element method for the Timoshenko beam associated with energy indicator to assess localization level of beam vibrations.

2. Timoshenko beam model and kinetic energy indicator of mode localization

Consider a micro-sized cantilever beam with total length L_t , composed of two segments made of two different materials. The first segment has length L_{b1} , elastic modulus E_1 , mass density ρ_1 ; and the second one has length L_{b2} , elastic modulus E_2 , mass density ρ_2 . We introduce two parameters $\alpha = E_2 / E_1$, $\beta = \rho_2 / \rho_1$ characterizing the ratios of the elastic moduli and mass densities of segment 2 to segment 1. The cross-sections of the two beam segments are assumed to be the same, $A = b \times h$, where b is the width and h is the thickness of the beam. The coordinate system of the beam is chosen such that the x-axis is directed along the length of the beam, the z-axis is directed downwards and perpendicular to the x-axis, as shown in Fig. 1.

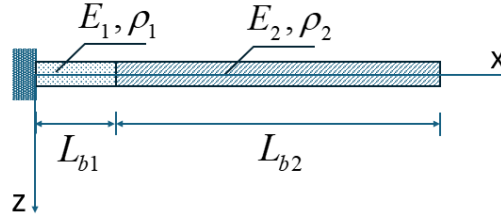


Figure 1 – Timoshenko micro-sized beam model with two segments made of two different materials

Within the framework of this research, Timoshenko's beam theory is used to investigate the vibration characteristics of the beam. Let $u_x^{(i)}(x, z, t)$, $u_z^{(i)}(x, z, t)$ be displacement fields in x , z -directions of the i -th segment where the index $i=1$ shows the first one with $0 \leq x \leq L_{b1}$, and the index $i=2$ presents the second one with $L_{b1} \leq x \leq L_t$ along beam's axis. The fields $u_x^{(i)}(x, z, t)$, $u_z^{(i)}(x, z, t)$ can be represented as follows [17]:

$$\begin{aligned} u_x^{(i)}(x, z, t) &= -z\theta_i(x, t) \\ u_z^{(i)}(x, z, t) &= w_i(x, t) \end{aligned} \quad (1)$$

where $\theta_i(x, t)$, $w_i(x, t)$ are rotation and deflection of beam axis, respectively. In each beam segment, deflection and rotation angle are considered as two independent variables. In Euler-Bernoulli beam theory, the rotation angle is approximated as the derivative of deflection. For free vibration, we assume that $w_i(x, t)$, $\theta_i(x, t)$ ($i=1, 2$) can be represented via the modal shape functions $W_i(x)$, $\Theta_i(x)$ as follows:

$$w_i(x, t) = W_i(x) \cos(\omega t), \quad \theta_i(x, t) = \Theta_i(x) \cos(\omega t) \quad (2)$$

where ω is the natural vibration frequency of the beam. The mode properties may be revealed through the distribution of vibrational energy in the two beam segments. Kinetic energy is one of the important indicators to be considered when determining which beam segment is dominant; this property is referenced as an indicator of energy localization. Therefore, vibration localization of the beam will be calculated through the kinetic energy ratio between two segments with various system parameters. The kinetic energy of the beam is the total kinetic energy of the two beam segments:

$$T = T_{b1} + T_{s1} + T_{b2} + T_{s2} \quad (3)$$

where T_{b1} , T_{s1} , T_{b2} , T_{s2} are kinetic energy contributions due to bending and shearing of segments 1 and 2, respectively:

$$\begin{aligned} T_{b1} &= \frac{1}{2} \rho_1 A \omega^2 \sin^2(\omega t) \int_0^{L_{b1}} W_1^2(x) dx, \quad T_{s1} = \frac{1}{2} \rho_1 I_y \omega^2 \sin^2(\omega t) \int_0^{L_{b1}} \Theta_1^2(x) dx \\ T_{b2} &= \frac{1}{2} \rho_2 A \omega^2 \sin^2(\omega t) \int_{L_{b1}}^{L_t} W_2^2(x) dx, \quad T_{s2} = \frac{1}{2} \rho_2 I_y \omega^2 \sin^2(\omega t) \int_{L_{b1}}^{L_t} \Theta_2^2(x) dx \end{aligned} \quad (4)$$

For the Timoshenko beam model under consideration, the ratio η_{TB} of the kinetic energy of segment 2 to segment 1 is introduced:

$$\eta_{TB} = \frac{T_{b2} + T_{s2}}{T_{b1} + T_{s1}} = \beta \frac{\int_{L_{b1}}^{L_t} W_2^2(x) dx + r_y^2 \int_{L_{b1}}^{L_t} \Theta_2^2(x) dx}{\int_0^{L_{b1}} W_1^2(x) dx + r_y^2 \int_0^{L_{b1}} \Theta_1^2(x) dx} \quad (5)$$

where $r_y = \sqrt{I_y / A}$ is radius of gyration of beam's cross section. The ratio η_{TB} is considered as an indicator to assess the degree of localization of vibration energy on the two beam segments. In the case of the Euler-Bernoulli beam, because shear strain is not considered in Eq. (5), the kinetic energy component due to shear strain will be ignored. As a result, we obtain the energy ratio of segment 2 to segment 1 for the Euler-Bernoulli beam as follows:

$$\eta_{EB} = \beta \frac{\int_{L_{b1}}^{L_1} W_2^2(x) dx}{\int_0^{L_{b1}} W_1^2(x) dx} \quad (6)$$

where the finite integrals are calculated on two different beam segments, in which the integrands are functions corresponding to the j -th modal shapes.

3. Free vibration equation of beam using the finite element method

Timoshenko's beam theory is known as an extension of Euler-Bernoulli beam theory. The number of degrees of freedom in the equation of motion, thus, increases compared to the Euler-Bernoulli beam. The beam segments 1 and 2 are divided into n_1 , n_2 elements, respectively. The nodes are numbered from 1 to $n_1 + n_2 + 1$, where segment 1 is numbered from 1 to $n_1 + 1$ and segment 2 is taken from $n_1 + 1$ to $n_1 + n_2 + 1$. Assume that the elements on each beam segment are of the same length. For beam segment 1, the element length is $L_1 = L_{b1} / n_1$. The element length in segment 2 is $L_2 = L_{b2} / n_2$. The degrees of freedom of each element in each segment are illustrated in Figure 2. The element's nodal displacement vector is denoted as $\{q\}_e = \{w_e \ \theta_e \ w_{e+1} \ \theta_{e+1}\}^T$ ($e = 1, 2, \dots, n_1 + n_2$). Combined nodal displacement vector in the global coordinate is $\{q\} = \{w_1 \ \theta_1 \ w_2 \ \theta_2 \ \dots \ w_{n_1+n_2} \ \theta_{n_1+n_2} \ w_{n_1+n_2+1} \ \theta_{n_1+n_2+1}\}^T$ for which the boundary between two segments is a common node.

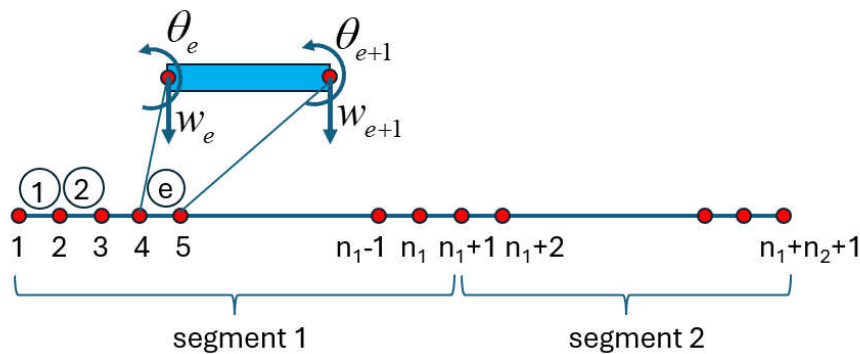


Figure 2 – Illustration of degree of freedoms of elements and numbered nodes along the length of the beam

Using the standard finite element method for the Timoshenko beam with exact shape functions found from the governing equation of free vibration of beam [18], we obtain an equation of motion for the beam through the degrees of freedom of the nodes as follows:

$$[M]\{\ddot{q}\} + [K]\{q\} = \{0\} \quad (7)$$

where $[M]$, $[K]$ are global mass and stiffness matrices after assembling element's mass and stiffness

matrices, respectively. The size of these matrices is $2(n_1 + n_2 + 1) \times 2(n_1 + n_2 + 1)$. Because the beam's boundary is clamped at node 1, the deflection w_1 and rotation angle θ_1 are both zeros. By imposing this condition, we obtain the following reduced equation of motion:

$$[\bar{M}]\{\ddot{\bar{q}}\} + [\bar{K}]\{\bar{q}\} = \{0\} \quad (8)$$

where $[\bar{M}]$, $[\bar{K}]$ are the reduced global mass and stiffness matrices with size $2(n_1 + n_2) \times 2(n_1 + n_2)$; $\{\bar{q}\}$ is a reduced global nodal displacement vector after removing the degree of freedom of the boundary node 1. Assume that the vector $\{\bar{q}\}$ can be represented as a solution of periodic form in time:

$$\{\bar{q}\} = \{\bar{q}_0\} \cos(\omega t) \quad (9)$$

where $\{\bar{q}_0\}$ is a amplitude vector. By substitution of solution (9) into Eq. (8) yields the following homogeneous algebraic equation for $\{\bar{q}\}$:

$$([\bar{K}] - \omega^2 [\bar{M}])\{\bar{q}\} = \{0\} \quad (10)$$

Eq. (10) can be transformed into the following equivalent representation:

$$([\bar{M}]^{-1} [\bar{K}] - \omega^2 [I]_{2(n_1+n_2)})\{\bar{q}\} = \{0\} \quad (11)$$

where the notation $[\bar{M}]^{-1}$ is an inverse matrix of the mass matrix $[\bar{M}]$, $[I]_{2(n_1+n_2)}$ is a unit matrix with size $2(n_1 + n_2) \times 2(n_1 + n_2)$. The Eq. (11) shows that the quantity ω^2 is a eigenvalue of the squared matrix $[\bar{M}]^{-1} [\bar{K}]$ and is the solution of the following characteristic equation:

$$\det([\bar{M}]^{-1} [\bar{K}] - \omega^2 [I]_{2(n_1+n_2)}) = 0 \quad (12)$$

By solving the problem of eigenvalue and eigenvector of the matrix $[\bar{M}]^{-1} [\bar{K}]$, we obtain a vector of set of frequencies $\{\omega_1 \ \omega_2 \ \dots \ \omega_{2(n_1+n_2)}\}^T$ with $2(n_1 + n_2)$ elements and a set of eigenvectors $\{V_i\}_{i=1,2,\dots,2(n_1+n_2)}$. The eigenvector $\{V_i\}$ is separated into a vector $\{W^{(i)}\}$ corresponding to the deflection and a vector $\{\Theta^{(i)}\}$ corresponding to the rotation angle of the beam. The vector $\{W^{(i)}\}$ has elements located at odd-index positions in the eigenvector $\{V_i\}$, while the vector $\{\Theta^{(i)}\}$ has elements located at even-index positions in the eigenvector $\{V_i\}$. In this study, we will investigate the behavior of the vibration modes to clarify the localized nature of the vibration of the two-segment cantilever beam under consideration based on the characteristics of the vibration energy of the modes.

4. Numerical results and discussions

In this subsection, we consider a cantilever beam with total length $L_t = 460 \ \mu\text{m}$ and cross-sectional dimension $b \times h = 49 \ \mu\text{m} \times 2.3 \ \mu\text{m}$. The first segment of the beam is made of silicon material with elastic

modulus $E_1 = 130$ GPa, mass density $\rho_1 = 2329$ kg/m³, and Poisson's ratio $\mu_1 = 0.28$. (For silicon material, the elastic modulus E_1 is taken in the (100) direction in the crystal structure). The second segment is made of silicon carbide, abbreviated as SiC, which has a larger elastic modulus than the silicon material. In our calculations, we use coefficient value $\alpha = 2.66$, i.e. $E_2 = \alpha E_1 = 345.8$ GPa; coefficient value $\beta = 1.66$, i.e. $\rho_2 = \beta \rho_1 = 1.38 \times 2329 = 3214.02$ kg/m³. Poisson's ratio of the SiC material is taken as $\mu_2 = 0.19$. In case $\alpha = 1$ and $\beta = 1$, we obtain homogeneous material for the two segments of the beam, i.e. the two-segment beam problem becomes a homogeneous beam problem with a single material. Table 1 shows that the first and second vibration frequencies calculated using the Timoshenko beam FEM model and Euler-Bernoulli beam FEM model are in a good agreement with those obtained from exact solutions [20] in previous publications, and experimentally measured frequencies [19]. Because the considered beam is relatively slender and thin, the results of Timoshenko's beam theory and Euler-Bernoulli's beam theory differ only slightly. The difference increases if the beam length is reduced and the beam thickness is increased.

Five modal shapes of vibration of the considered micro-cantilever with two material segments are illustrated in Fig. 3 for beam deflection. The vibration modes will be normalized according to the mass of a reference beam of length L_t , mass density ρ_1 , i.e. $\{V_i\}^T [\bar{M}] \{V_i\} = m_{ref}$ with $m_{ref} = \rho_1 A L_t$.

Table 1

Comparison of the first and second vibration frequencies in the case of a homogeneous beam material using different approaches

	Experiment [19] (kHz)	Analytical [20] (kHz)	EB-FEM (kHz)	TB-Present (kHz)
1st freq.	12.80	13.1184	13.1184	13.1181
2nd freq.	78.78	82.2113	82.2113	82.1996

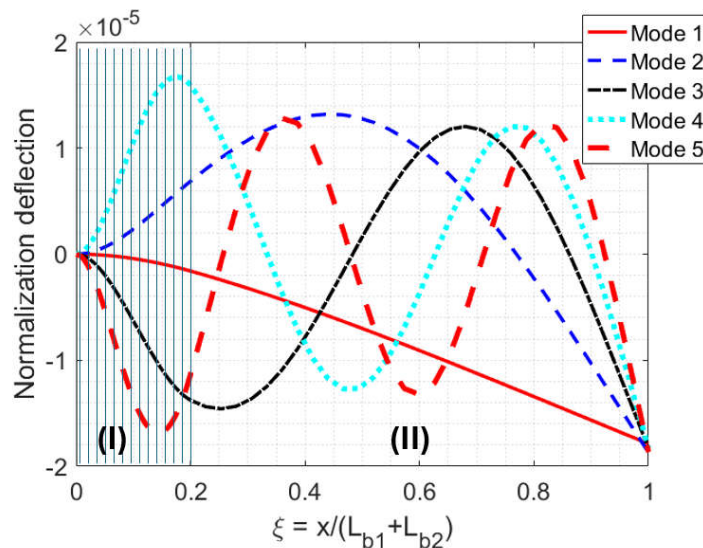


Figure 3 – The first five eigenmodes of deflection in cantilever beam vibration. (I): the first segment; (II): the second segment

In Fig. 3, along the length of beam, two regions are separated, one presents for the first segment, denoted as region (I) with $0 \leq \xi = x / L_t \leq \xi_1 = L_{b1} / L_t$; the second region (II) with

$\xi_1 = L_{b1} / L_t \leq \xi = x / L_t \leq 1$; ξ_1 is taken as $\xi_1 = 0.2$. The first vibration mode has the greatest deflection at the free end of the beam. The first mode has no antinodes, while modes 2, 3, 4, 5 have 1, 2, 3, 4 antinodes respectively. Modes with antinodes will have zero-points at which the eigenvalue is zero. This is known similarly to the free vibration of a conventional cantilever beam with a homogeneous material. The difference here compared to a homogeneous material beam is that the influence of the two beam materials leads to different energy distribution on the two beam segments. With parameters $\alpha = 2.66$, $\beta = 1.38$, they show that segment 1 has a much "softer" elastic modulus than segment 2, and a lower mass density than segment 2; thus, for a cantilever beam, the vibration properties will be skewed towards one of the two segments. Physically, based on the indicator in equation (5), energy localization will appear on the two beam segments where one segment will occupy a significantly energy level than the other.

In Fig. 4, for the same elastic modulus, increasing the mass density ratio β leads to a decrease in first frequency of the beam. In other words, increasing the density of the second segment has a softening effect on the structure, making it more susceptible to low-frequency vibrations in the range of approximately 5.5 kHz to 17.5 kHz. The position at which $\beta = 1$ shows that the mass density of the two beam segments is the same. If parameter β is much smaller than 1, the beam will vibrate at a relatively high frequency, and the highest frequency occurs when the ratio β is taken as the lower bound, i.e. $\beta = 0.4$, within the considered range.

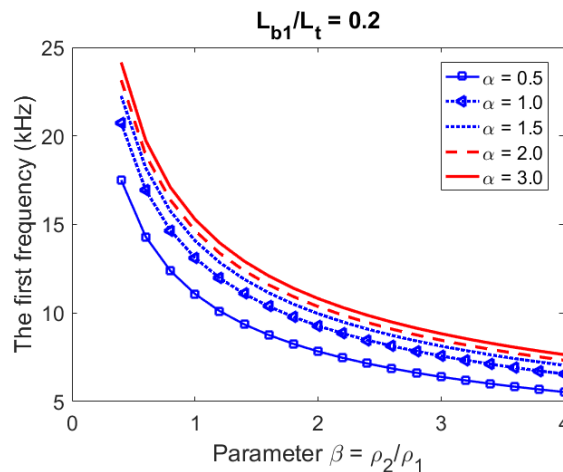


Figure 4 – Curves of the first frequency depending on the ratio $\beta = \rho_2 / \rho_1$ with different values of parameter α and with the ratio $L_{b1} / L_t = 0.2$

On the other hand, with the same mass density ratio, increasing the ratio α will increase the first vibration frequency, that means the beam will become stiffer. In Fig. 4, the value $\alpha = 1$ shows the frequency curve of the beam with two segments having the same Young's modulus. Thus, two opposite trends can be observed. On the one hand, increasing the mass density reduces the frequency of vibrations of the system, indicating a softening effect. On the other hand, increasing Young's modulus ratio raises the natural frequencies, reflecting a stiffening effect. These completing influences may be referred to as a softening-stiffening effect associated with variations in beam material properties. Specially, an increase in mass density ratio softens the dynamic response of the beam, whereas an increase in Young's modulus ratio stiffens it. Such completing effects are absent in homogeneous beams where the material properties remain uniform throughout the structure.

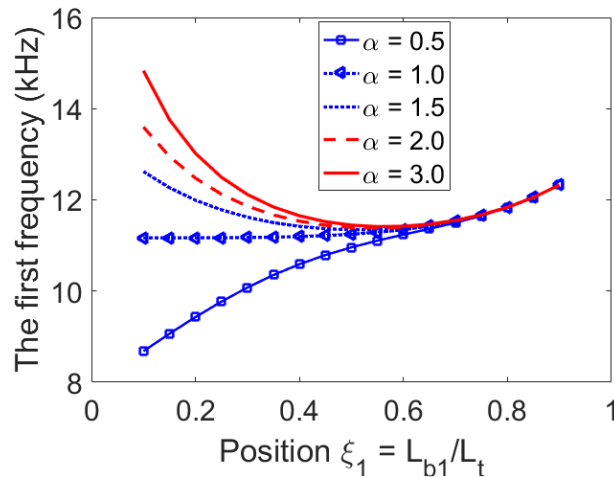


Figure 5 – Curves of the first frequency depending on beam length ratio $\xi_1 = L_{b1} / L_t$ with different values of modulus ratio $\alpha = E_2 / E_1$ and with the ratio $\beta = 1.38$

Vibration localization properties can be revealed through the frequency distribution when changing the common boundary point between two beam segments, i.e., changing the ratio of the length of the first segment to the total beam length. In Fig. 5, the first frequency has a relatively different distribution along the beam length when changing the ratio parameter α from 0.5 to 3.0. For the considered parameters, when $\alpha = 1.5, 2.0, 3.0$, the first frequency even has an extreme point at a specified position along the beam length. This means that there exists a position corresponding to the ratio $\xi_1 = L_{b1} / L_t$ such that the first frequency of the beam reaches its minimum value, i.e., the beam is in a relatively "soft" state compared to other positions. This is very useful in designing highly sensitive sensors based on frequency distribution. This characteristic is a result of vibration localization when the beam material is no longer homogeneous. This suggests that it can provide an alternative design approach in comparison to the traditional sensor designs with homogeneous materials. As the ratio ξ_1 tends to 1, the frequency curves corresponding to the parameter α approach to each other's, that is, they are asymptotically close to the frequency value of the homogeneous material.

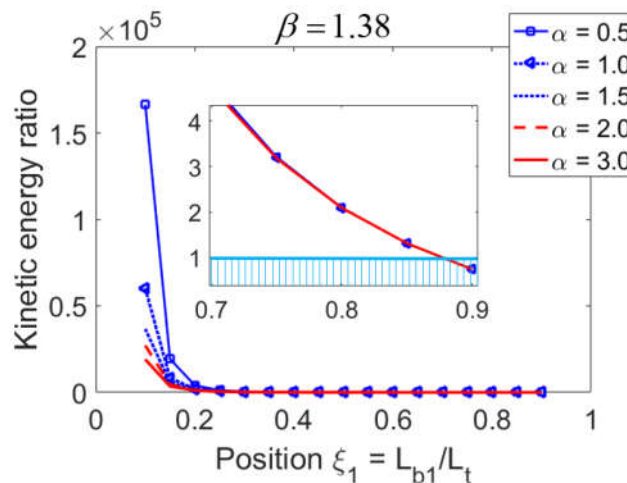


Figure 6 – Kinetic energy ratio between two segments depending on the beam length parameter $\xi_1 = L_{b1} / L_t$ with various values of modulus ratio $\alpha = E_2 / E_1$ and with the ratio $\beta = \rho_2 / \rho_1 = 1.38$

The localized nature of the vibration is more clearly reflected through the energy indicator η_{TB} . Fig. 6 illustrates the curves of the energy ratio changing with various location $\xi_1 = L_{b1} / L_t$ under different values of parameter α ; the value β is taken as $\beta = 1.38$. The emphasis here is on the value $\beta = 1.38$ leading to the fact that the ratio η_{TB} has a relatively large value when location ξ_1 close to 0, and almost on the interval $[0.1, 0.9]$ the value of η_{TB} is greater than 1; only when value ξ_1 is larger than about 0.87 the ratio η_{TB} is less than 1. This reflects that, with ξ_1 is less than 0.87, the kinetic energy is almost concentrated in segment 2, whereas if value ξ_1 is larger than 0.87, the kinetic energy gradually shifts to predominating in segment 1. This energy difference shows that the degree of localization is relatively clear in cantilever beams without homogeneous materials, for example, the material in segment 2 has a larger elastic modulus than segment 1.

In Fig. 7, when exploring the ratio η_{TB} with the value $\beta = 0.4$, the degree of energy localization is also relatively clear: energy is concentrated in segment 2 if ξ_1 is less than 0.75, while a reversal of energy concentration occurs if ξ_1 is greater than 0.75. The shift of ξ_1 from 0.87 to 0.75 is considered not large, however, this is a useful range to understand the reversal of energy concentration between the two beam segments when designing cantilever beam sensors with different parameters.

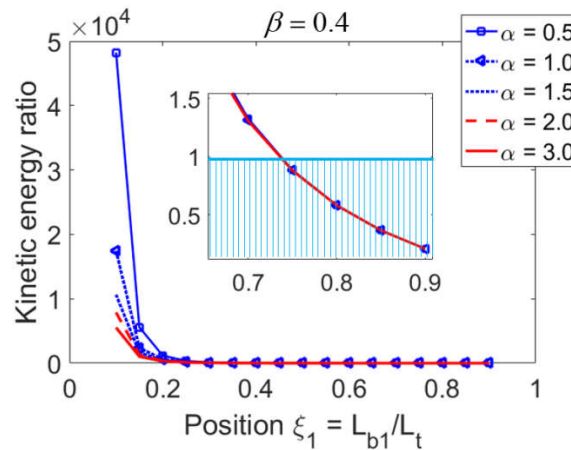


Figure 7 – Kinetic energy ratio between two beam segments depending on the parameter $\xi_1 = L_{b1} / L_t$ with various values of modulus ratio $\alpha = E_2 / E_1$ and with the ratio $\beta = \rho_2 / \rho_1 = 0.4$

5. Conclusions

Mode localization has emerged as an attractive sensing mechanism because it can achieve significantly higher sensitivity than conventional frequency-shift-based resonant sensors in many modern sensing applications. In this study, the authors obtained the following results:

- A kinetic energy-based approach is proposed to evaluate the degree of localized vibration of a cantilever microbeam with two different material segments.
- The study reveals a novel physical effect observed related to the change in vibration frequency of the beam when changing Young's modulus and mass density ratios of the two beam segments. The observed vibration properties are referenced as a stiffening-softening effect dependent on the beam material parameters.
- When changing Young's modulus parameter, there exists a beam configuration at which the first frequency can be minimized, meaning that beams can be designed with a specified ratio of the first segment

length to the total length to minimize the first frequency.

- It is shown that the concentration of kinetic energy is mainly located in the second segment of the beam, with the ratio of the first segment length to the total length being less than 0.75 to 0.87, depending on the value of Young's modulus ratio of the two beam materials.

In this study, the authors have used the Timoshenko beam model to evaluate the energy localization and vibration frequency of micro-sized beams and verified it against known results. However, at the micro-scale level, the Timoshenko beam model needs to consider the influence of the "internal size" parameter, i.e., the intrinsic micro-scale parameter of the beam, to achieve a more accurate quantitative assessment, helping to design more reliable sensors.

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ПОВЫШЕНИЕ ПРОИЗВОДИТЕЛЬНОСТИ ИНТЕРНЕТА ВЕЩЕЙ В БЕСПРОВОДНЫХ И МОБИЛЬНЫХ СЕТЯХ ЗА СЧЁТ ИНТЕГРАЦИИ ИМЕНОВАННЫХ СЕТЕЙ (NDN) И ГРАНИЧНЫХ ВЫЧИСЛЕНИЙ

Аннотация

Статья посвящена новой адаптивной структуре NDN-Edge Computing, которая динамически оптимизирует извлечение данных, кэширование и распределение вычислительных ресурсов

Ключевые слова:

интернет вещей, именованные сети (NDN), граничные вычисления.

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ENHANCING IOT PERFORMANCE IN WIRELESS AND MOBILE NETWORKS THROUGH NAMED DATA NETWORKING (NDN) AND EDGE COMPUTING INTEGRATION

Abstract

This paper proposes a novel adaptive NDN-Edge Computing framework that dynamically optimizes data retrieval, caching, and computational resource allocation.