

ORIGINAL ARTICLE

Differential Response of Fracture Characterization of Mode III Fracture in Sandstone Under Dynamic Versus Static Loading

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Received: 20 August 2024 | **Revised:** 18 October 2024 | **Accepted:** 28 November 2024

Funding: This study is financed by the National Natural Science Foundation of China (Nos. 52274100, 52179118, and 42077240).

Keywords: dynamic loading | loading rate | mode III fracture | morphology characteristics | static loading

ABSTRACT

This work examines the effect of loading rate (K) on the mode III fracture behavior of sandstone. Edge-notched diametrically compressed (ENDC) disc sandstone specimens were tested under different static and dynamic mode III fracture loadings, revealing a clear loading rate effect on both mode III and mode I fractures. Specifically, the peak load and fracture toughness (K_{IIC} , K_{IC}) increase as the K increases across both static and dynamic scales. At the static scale, the K_{IIC} is about 1.28–1.38 times of the K_{IC} , whereas at the dynamic scale, the K_{IIC} is less than the K_{IC} . The relationship between K_{IIC} and K_{IC} is affected by the loading scale and the shape of the specimen, but the data collected thus far indicate that the origin and type of rock have minimal effect on this relationship. In addition, the fracture surface morphology characteristics were quantitatively analyzed.

1 | Introduction

In underground engineering, rock fracture occurs constantly [1, 2]. Therefore, understanding the characteristics of rock fracture behavior is crucial, whether it is for support engineering or rock excavation projects [3–5]. The fracture behavior of rock has been extensively researched, with a focus on factors like high temperature [6, 7], moisture content [8, 9], anisotropy [10, 11], loading speed [12, 13], size effect [14, 15], prestress [16, 17], and strain rate [18, 19], and how they affect fracture behavior. According to the simple crack classification method proposed by Irwin [20], the objects of study for the above fracture behaviors are all in-plane cracks. In fact, the fracture behavior of natural rock mass under complex stress environment not only occurs in the plane [21]. However, there is relatively limited experimental

research on the characteristics of out-of-plane shear cracks in rock. For instance, Gan et al. [22] elucidated the mechanism by which acid corrosion affects the mixed mode I–III fracture behavior of sandstone, focusing on both chemical reactions and microstructural changes. Hua et al. [23] analyzed how the resistance of sandstone to mode III fractures deteriorates when subjected to cycles of wetting and drying in an acidic solution. Yang et al. [24] investigated how thermal effects influence the mixed mode I–III fracture behavior of diorite from the acoustic emission signal characteristics perspective. Shi et al. [25] investigated how thermal effects influence the pure mode III fracture behavior of granite based on fracture surface morphology characteristics. Bidadi, Akbaridoost, and Aliha [26] explored how the thickness of rock affects mode III crack propagation behavior and bearing capacity.

Summary

- Mode III fracture of sandstone at static and dynamic scale was performed, respectively.
- The effect of loading rate on the fracture characteristics of sandstone was discussed.
- The fitting formula of K_{IIC} and loading rate in static and dynamic range was proposed.
- The fracture surface morphology was quantitatively analyzed.

Besides the factors previously mentioned, such as acid corrosion, high-temperature thermal effect, and thickness effect that affect the behavior characteristics of mode III fracture in rocks in underground engineering, the heterogeneity of the rocks themselves leads to a significant loading rate (K) effect [27, 28]. Namely, rock specimens exhibit different mechanical, deformation, and fracture behavior characteristics under different loading conditions. Dynamic studies on the rock under hydrostatic pressure were reported by Du et al. [29], investigating the impact of high loading rates on rock deformation characteristics, failure modes, and energy distribution under different hydrostatic pressures. Oh et al. [30] carried out fracture toughness (K_{IC}) tests, both static and dynamic, on straight notched disc bend (SNDB) granite specimens to examine how microcracks contribute to anisotropy in fracture toughness and how this anisotropy is influenced by loading rate. Zhang and Zhao [31] observed that as the loading rate increases, the dynamic K_{IC} of marble also increases in a nearly linear manner. They also studied the microscopic failure mechanisms involved. Yan et al. [32] reported that a strong correlation between dynamic K_{IIC} and K , with transgranular cracks becoming increasingly dominant in dynamic mode II fracture process of sandstone as the loading rate rises. Zhang et al. [33] conducted dynamic mode II fracture tests using a modified short-core-in-compression with an enhanced Hopkinson bar system. Their findings reveal that dynamic K_{IIC} follows a power-law increase with the K , whereas the normalized dynamic K_{IIC} exhibits a liner increase with the normalized K . Li et al. [34] discussed the K dependency of dynamic mixed mode I–II fracture through numerical simulations. They found that dynamic K_{IC} varies more significantly than dynamic K_{IIC} when the K increases by the same amount. However, there is still limited research on how loading rate affects mode III fractures.

Therefore, in this work, mode III fracture tests were conducted on sandstone using the edge-notched diametrically compressed (ENDC) disc specimen and the mode III fracture testing method. By varying the loading speed (v) and impact pressure (p), mode III fracture behavior was examined at both static and dynamic scales. The influence of loading speed and impact pressure was investigated, and the effect of loading rate K on mode III fracture was analyzed by normalizing the loading rate. The relationship between K_{IIC} and K_{IC} was presented. Additionally, three-dimensional scanning technology was used to reconstruct the fracture surfaces, enabling a quantitative analysis of the morphological features of mode III fracture at different loading rate.

2 | Experimental Method

2.1 | Methodology of Mode III Fracture

Based on the loading conditions, the simple cracks are categorized into three modes, that is, opening mode (mode I), in-plane shear mode (mode II), and out-of-plane shear mode (mode III) (also known as out-of-plane tearing fracture) [20]. Various specimen shapes and test methods have been developed to examine the out-of-plane fracture behavior of rock, such as the edge-notched disc bend (ENDB) specimen [35], ENDC disc specimen [36], asymmetric disc bend (ADB) specimen [37], axially double-edge notched Brazilian disc (ANBD) specimen [38], and more [39, 40]. Among these, ENDC and ENDB specimens are extensively employed in mode III fracture loading tests of rock and asphalt materials for their simple geometry, easy fabrication, and mode III loading without the requirement of a special loading device. For example, based on the samples of these two shapes, scholars have proposed a mode III fracture toughness measurement method for concrete and asphalt materials [41–43] and carried out a study on the mode III fracture related behavior of self-compacting concrete, embedded aggregate concrete, cement emulsified asphalt mortar, and other materials [44–48]. According to the research conducted by Bahmani et al. [36], it was found that ENDC specimens used to measure mode III fracture have higher reliability. The most reliable measurement point in ENDB specimens is at the crack front midpoint ($Z/R=0$). In contrast, in ENDC specimens, the impact of mode II is relatively minimal within the range of $|Z/R| \leq 0.5$. Hence, ENDC specimens were selected for loading tests in this work, which was achieved by adjusting the loading angle to implement mode I and mode III loading.

An ENDC specimen with a diameter of $2R$ and a thickness of B , as depicted in Figure 1, features an edge notch oriented along of the disc diameter. The length of the notch along the thickness of the disc is denoted as a . A linear load (P) is applied to the specimen, where the loading direction forms an angle (β) with the notch plane. The stress intensity factors (SIFs) K_I and K_{III} for the ENDC specimen can be expressed as [36]

$$K_I = \frac{P\sqrt{\pi a}}{RB} Y_I \quad (1)$$

$$K_{III} = \frac{P\sqrt{\pi a}}{RB} Y_{III} \quad (2)$$

K_I and K_{III} represent the SIF for mode I and mode III, respectively. Y_I and Y_{III} are the associated geometrical factors for mode I and mode III, which depend on the ratio a/B and the angle β . Bahmani et al. [36] employed finite element numerical simulation software to estimate Y_I and Y_{III} for an ENDC specimen where $a/B=0.4$: When β is 0° , the specimen exhibits pure mode I fracture ($Y_I=0.65$, $Y_{III}=0$); when β is 18° , the specimen exhibits pure mode III fracture ($Y_I=0$, $Y_{III}=0.654$). When P reaches a critical value of P_0 , the ENDC specimen experiences failure. In this case, the K_C value is equal to the fracture toughness of the specimen.

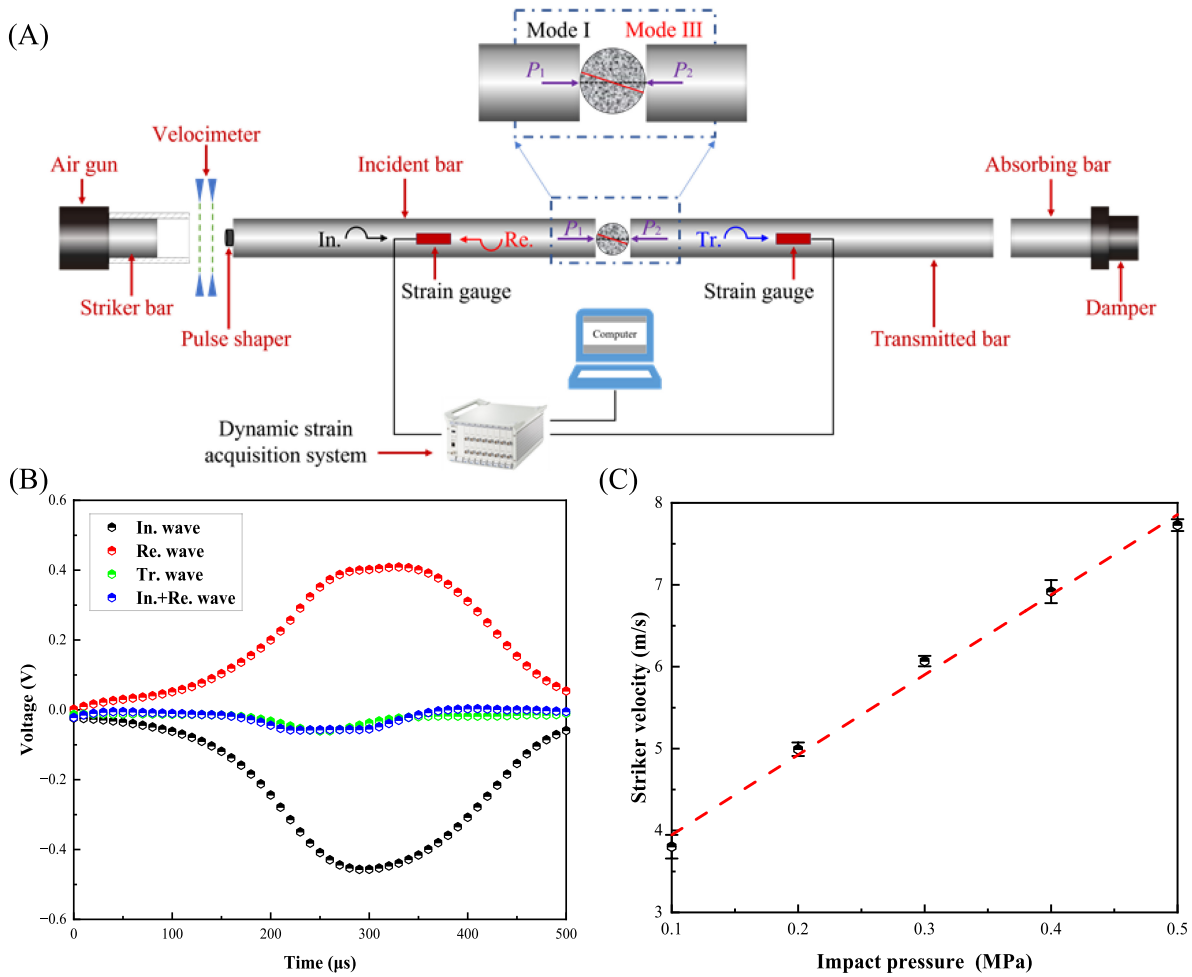


FIGURE 3 | (A) Schematic diagram of SHPB testing system. (B) Typical dynamic force balance check (In. wave, the incident wave; Re. wave, the reflected wave; Tr., the transmitted wave; In. + Re. wave, the superposition of incident and reflected wave). (C) Striker velocity versus impact pressure. [Colour figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com)]

or 18°) for both loading modes. To eliminate end-face friction effects, a thin layer of Vaseline was applied at the contact points between the specimen and the bars. To generate an In. stress wave, the air pressure was regulated to drive the 50 mm diameter, 400 mm long striker against the In. bar. A rubber washer (10 mm in diameter and 2 mm thick) was positioned at the center of the In. bar as a pulse shaper to enhance the shape of the incident wave and maintain a consistent strain rate loading. When the striker impacts the In. bar, the resulting In. stress wave travels along the bar, resulting a reflected (Re.) wave on the In. bar and a Tr. wave on the Tr. bar after interacting with the specimen. These stress waves were captured by strain gauges with a sensitivity of 100 attached to the bars and collected as electrical signals by a dynamic strain acquisition device.

The data from the dynamic strain acquisition device were processed to noise reduction, resulting in the typical waves of In., Re., and Tr., as shown in Figure 3B. The waveform of the Tr. wave closely matches that of the In. + Re. wave, indicating that the specimen experiences dynamic force equilibrium and that the test data are reliable and effective. The impact velocities of the striker driven by different impact pressures were measured and found to be stable under each air pressure,

further proving the reliability of the test system, as depicted in Figure 3C.

The signals collected by the dynamic strain acquisition device were processed through the three-wave method to determine the dynamic load applied to the specimen. The load at both ends of the specimen was obtained using Equations (3) and (4) [50]. Based on the stress homogenization assumption, Equation (5) was utilized to calculate the average value of the loading force $P(t)$ [50].

$$P_1(t) = AE[\epsilon_1(t) + \epsilon_R(t)] \quad (3)$$

$$P_2(t) = AE\epsilon_T(t) \quad (4)$$

$$P(t) = \frac{EA}{2} [\epsilon_1(t) + \epsilon_R(t) + \epsilon_T(t)] \quad (5)$$

in which $P_1(t)$ and $P_2(t)$ represent the time-dependent dynamic forces at the ends of the specimen on the In. bar and the Tr. bar, respectively. A and E denote the cross-sectional area and elastic modulus of the bars. Furthermore, by substituting the peak dynamic loading of the specimen into Equations (1) and (2), the dynamic fracture toughness of the specimen is obtained.

3 | Result Analysis

3.1 | Static Test: Pure Mode III Fracture Properties of ENDC Sandstone Specimens Under Different Loading Speeds

Figure 4 illustrates the load–displacement curves derived from static loading tests for typical ENDC specimens in each group. The curve shapes are similar under different loading speeds and loading modes. Similar to the conventional loading and failure process for most rock specimens, the curves can be divided into four stages, that is, the compaction stage, the elastic deformation stage, the crack initiation and propagation stage (elastic–plastic deformation stage), and the post-peak failure stage [51, 52].

During the compaction stage, mode III fracture-loaded specimens exhibit longer and more pronounced compaction curves compared to mode I fracture-loaded specimens. Additionally, specimens loaded at slower speeds have more pronounced and longer prolonged compaction stages than those loaded at faster speeds. These differences in compaction stages are likely influenced by two factors. First, the change in loading mode results in a reduction in the vertical compaction force component. Mode III fracture loading tends to cause particles slippage around the

prefabricated notch, slowing down the compaction of the specimen. Second, at different loading speeds, higher loading speeds restrain particle slip and enhance interparticle compression. As a result, primary fine cracks are compacted more rapidly.

Similarly, the loading mode also affects the elastic stage. Uniaxial compaction tests reveal an apparent loading speed effect on the elastic modulus (the slope of the stress–strain curve in the elastic deformation stage) [53]. This phenomenon, where the mechanical and deformation properties of specimens vary due to loading speed, is also observed in the mode I loading process of ENDC specimens: The ability of the specimens to resist deformation (the slope of the load–displacement curve in the linear segment) increases with the increasing loading speed. However, unlike conventional uniaxial compression tests, this effect is less pronounced under mode III fracture loading.

To evaluate the effect of loading speed (v) on static mode III fracture loading, a comparison was made of the peak load (P_0) of the specimens. Figure 5 illustrates how the P_0 of the specimens change with loading speed under static loading. It is evident that the static P_0 of ENDC sandstone specimens are significantly influenced by the v . The P_0 increases as the v rises. In Figure 5A, as the v increases from 0.0001 to 0.001 mm/s, the average P_0 under

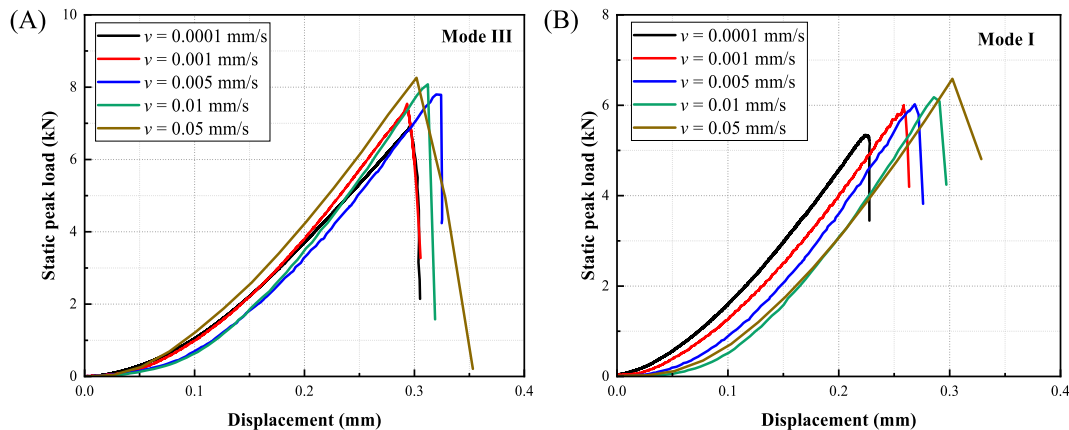


FIGURE 4 | Typical static load versus displacement curves of ENDC sandstone specimens with different loading speeds: (A) pure mode III and (B) pure mode I. [Colour figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com)]

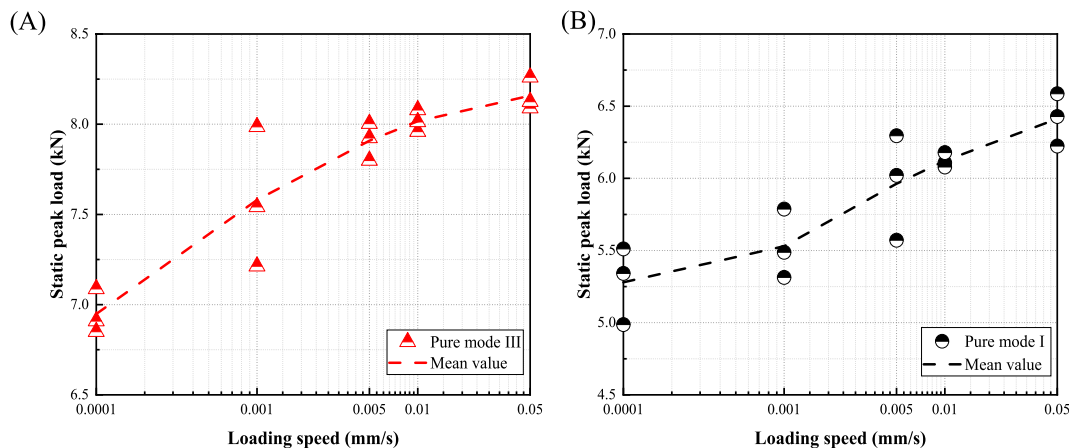


FIGURE 5 | Variation in peak load of ENDC sandstone specimen under static loading with different loading speeds: (A) pure mode III and (B) pure mode I. [Colour figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com)]

mode III loading rises from 6.95 to 7.58 kN, representing a 9.06% increase. When the v increases from 0.001 to 0.01 mm/s, the average P_0 under mode III loading further reaches to 7.91 kN, indicating a 4.34% increase. Similarly, the P_0 for mode I fracture loading increases with the v , as shown in Figure 5B.

The effect of loading speed is also reflected in the K_C of the specimens. Figure 6 illustrates how loading speed correlates with K_C of ENDC sandstone specimens under static loading. The K_{IIC} and K_{IC} of specimens increase with loading speed. When the loading speed increases from 0.0001 to 0.05 mm/s, the K_C of mode III fracture-loaded specimens and mode I fracture-loaded specimens increases by 17.40% and 21.46%, respectively. Additionally, compared to mode I fracture-loaded specimens, mode III fracture-loaded specimens exhibit higher static fracture toughness, approximately 1.28–1.38 times that of K_{IC} . For instance, at a loading speed of 0.005 mm/s, the average static K_{IIC} is 1.467 MPa·m^{0.5}, whereas the average K_{IC} is 1.099 MPa·m^{0.5}, resulting in the K_{IIC} 1.33 times that of K_{IC} .

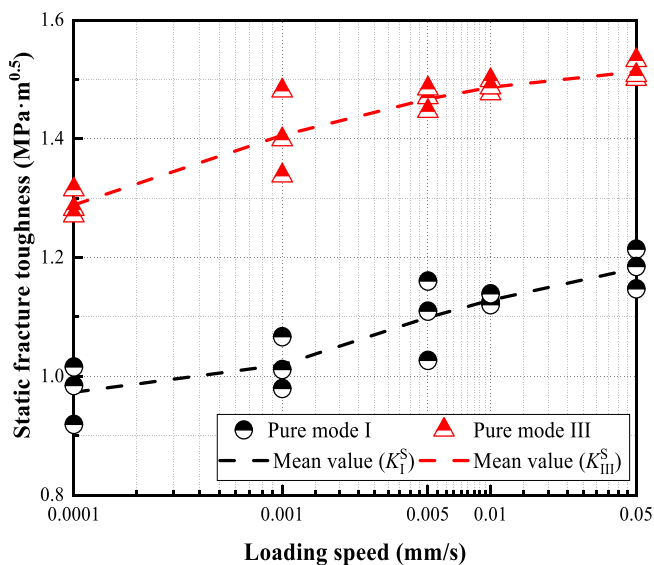


FIGURE 6 | Variation in fracture toughness of ENDC sandstone specimen under static loading with different loading speeds. [Colour figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com)]

3.2 | Dynamic Test: Pure Mode III Fracture Properties of ENDC Sandstone Specimens Under Different Impact Pressures

In real-world engineering contexts, rocks frequently encounter dynamic loads, including mechanical disturbances, seismic waves, and explosive shock waves. To examine the fracture behavior characteristics of out-of-plane tearing fractures under dynamic loading, impact tests on ENDC sandstone specimens were conducted at different impact pressures. Figure 7 shows typical load–time curves of ENDC sandstone specimens under different impact pressures. The curves of both mode III and mode I fracture-loaded specimens exhibit similar patterns. With an increase in impact pressure, the time required for specimen failure decreases. Additionally, the impact pressure influences the peak load of the specimens.

Figure 8 illustrates the relationship between the dynamic P_0 and impact pressure of ENDC sandstone specimens under dynamic loading. As an increase in impact pressure, the dynamic P_0 of the specimens show a growing trend. With the impact pressure increases from 0.1 to 0.3 MPa, the average dynamic P_0 of mode III fracture-loaded specimens rises from 21.11 to 29.23 kN, an increase of 38.47%. For mode I fracture-loaded specimens, the dynamic P_0 rises from an average of 22.22–31.09 kN, indicating a 39.92% increase. An increase in impact pressure from 0.3 to 0.5 MPa results in a 9.13% rise in the average dynamic P_0 for mode III fracture-loaded specimens. In contrast, the average dynamic P_0 increases by 5.21% under mode I fracture loading.

In addition to affecting the strength characteristics of the specimens, the impact pressure also influences the fracture properties. Figure 9 shows that both K_{IIC} and K_{IC} increase with increasing impact pressure, though the rate of increase diminishes as the impact pressure continues to rise. When the impact pressure rises from 0.1 to 0.3 MPa, the average value of K_{IIC} climbs from 3.916 to 5.422 MPa·m^{0.5}, representing a 38.46% increase. The average value of K_{IC} from 4.096 to 5.731 MPa·m^{0.5}, indicating a 39.92% increase. When the impact pressure increases from 0.3 to 0.5 MPa, the respective increases in K_{IIC} and K_{IC} are 9.13% and 5.20%.

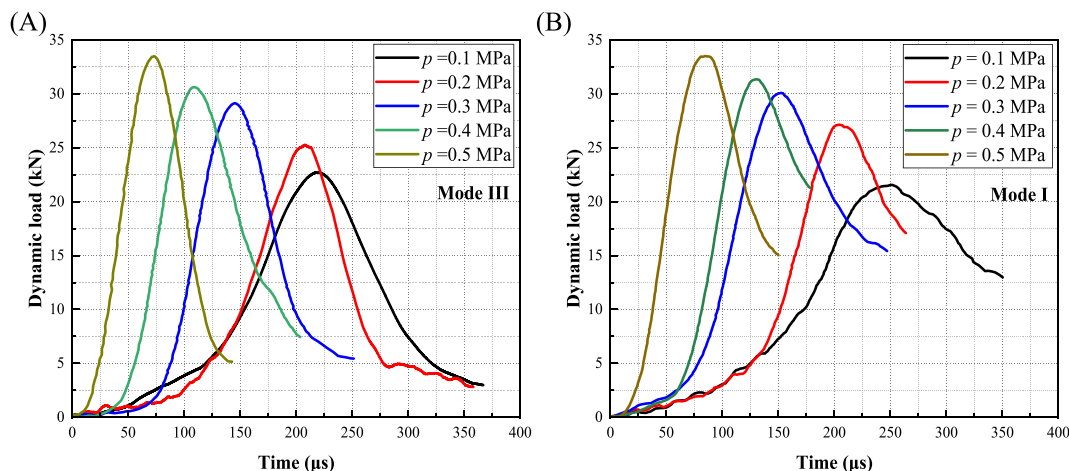


FIGURE 7 | Typical dynamic load versus time curves of ENDC sandstone specimens with different impact pressures: (A) pure mode III and (B) pure mode I. [Colour figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com)]

4 | Discussion

4.1 | Effect of Loading Rate on Pure Mode III Fracture Properties of ENDC Sandstone Specimens

In Sections 3, the strength and fracture characteristic responses of ENDC sandstone specimens under static and dynamic mode III fracture loading were discussed, respectively. Different “extents” of loading were achieved by controlling the loading speed and impact pressure in an attempt to analyze the strength and fracture characteristic of specimens under different “levels” of loading. To describe these “extents” of loading, scholars usually use strain rate [54–56] and loading rate (K) indicators to evaluate. Because the stress and strain distribution inside ENDC specimen are not uniformly distributed along the loading direction, it is difficult to apply the strain rate index. Therefore, the K is used in this paper. The loading rate (K) is determined by the slope of the SIF–time curve, as shown in Figure 10A,B.

In Figure 10C, it can be observed that the loading rates of mode III and mode I fracture-loaded specimens closely match under static loading. The loading rates (K) all tend to increase linearly with the increasing loading speed (v). Overall, the specimens under mode III fracture loading have a higher loading rate compared to mode I fracture loading, but the difference is not significant. Under dynamic loading, the loading rates of both mode III and mode I fracture-loaded specimens increase with higher impact pressure, and the difference in loading rates between the two becomes more pronounced (in Figure 10D). Notably, the K_I is higher than that of mode III under the same impact pressure. This difference may mainly arise because, under static loading, the intergranular cracks in the mode III fracture-loaded specimens are given enough opportunity to propagate fully. The intergranular cracks propagate in-plane in the loading direction and simultaneously extend out-of-plane, causing the stress concentration at the main crack front in mode III fracture specimens to occur more quickly under static loading. Conversely, under dynamic loading, the specimen tends to dissipate energy rapidly with transgranular cracks. The mode III fracture are difficult to expand rapidly, whereas the mode I fracture propagate relatively

quickly. The fracture surface of mode III fracture-loaded specimens contains more transgranular cracks, resulting in a high energy release rate, that is, high elastic potential energy and crack surface energy released per unit time. Consequently, the stress concentration at the main crack front of the mode I fracture-loaded specimen under dynamic loading occurs more rapidly.

Figure 11 illustrates the correlation between K_C and K of mode III fracture-loaded and mode I fracture-loaded specimens under static and dynamic loading tests. It is observed that the K_C of specimens increases with higher K under both dynamic loading and static loading.

In static loading tests, the mode III fracture-loaded specimens demonstrate significantly higher K_C than mode I fracture-loaded specimens, despite having only slightly higher K . In dynamic loading tests, the K of mode I fracture-loaded

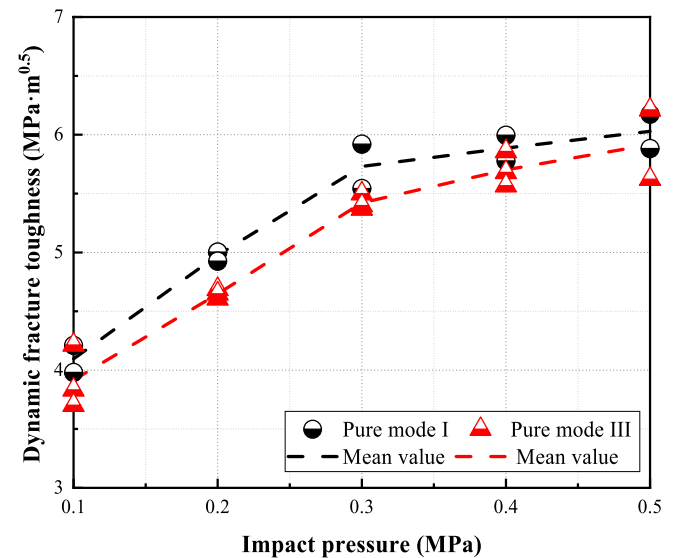


FIGURE 9 | Variation in fracture toughness of ENDC sandstone specimen under dynamic loading with different impact pressures. [Colour figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com)]

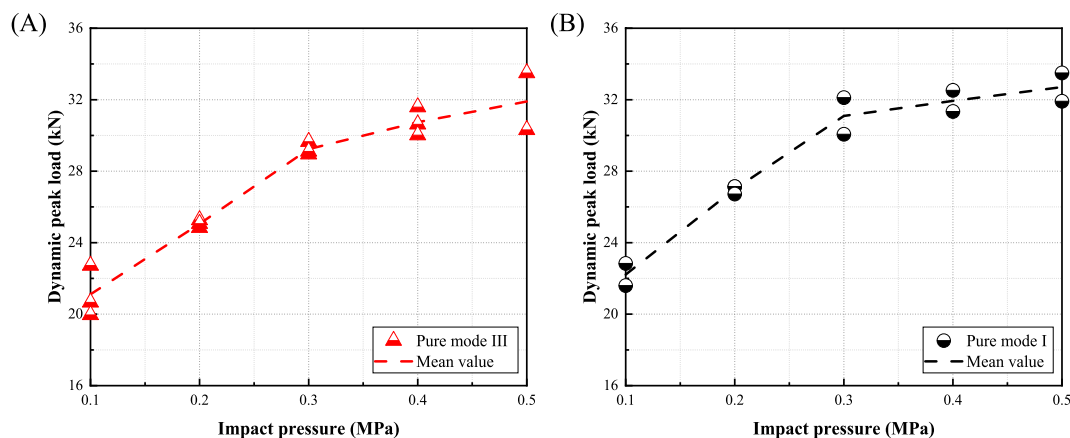


FIGURE 8 | Variation in peak load of ENDC sandstone specimen under dynamic loading with different impact pressures: (A) pure mode III and (B) pure mode I. [Colour figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com)]

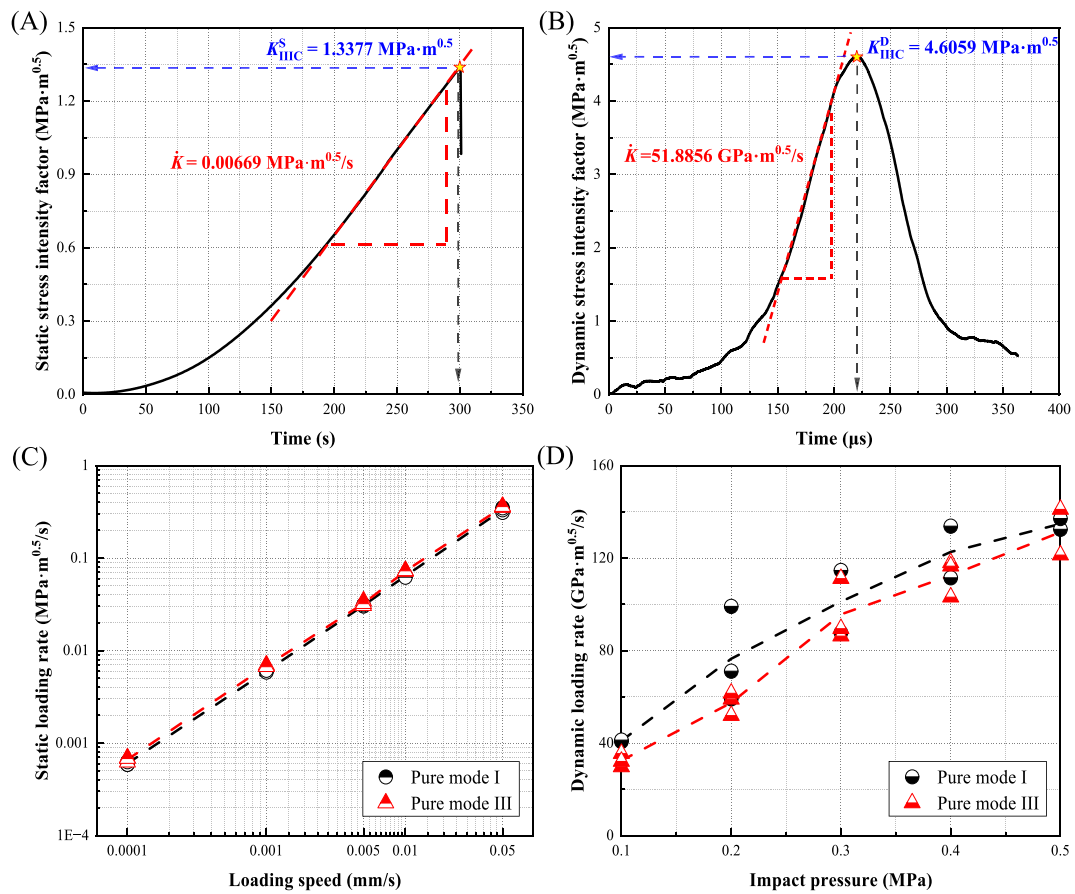


FIGURE 10 | ENDC sandstone sample. Loading rate determination method (A, typical static test; B, typical dynamic test) and loading rate test trend (C, loading speed vs. static loading rate [static test]; D, impact pressure vs. dynamic loading rate [dynamic test]). [Colour figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com)]

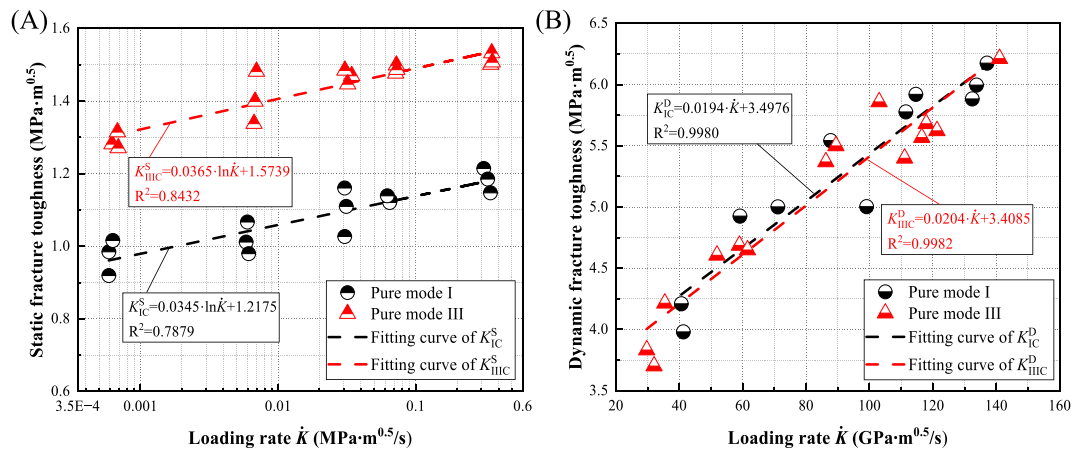


FIGURE 11 | Loading rate versus fracture toughness of ENDC sandstone sample: (A) static test and (B) dynamic test. [Colour figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com)]

specimens is obviously higher than that of mode III fracture-loaded specimens under the same impact pressure. However, the difference in K_C between the two loading modes is insignificant, and this difference primarily results from the varying duration of the linear increase stage in K_C under dynamic and static loading conditions (essentially coinciding with the elastic stage, where elastic deformation occurs). Under dynamic loading, the specimen failed before the crack

propagated utterly. To describe the changing trend, the K_C and K under static loading and dynamic loading were fitted by the previous correlation fitting formulas [30, 32, 57], and the consistency was well. Nevertheless, these fitting formulas used can only be applied to a certain range (static or dynamic) and are not suitable for predicting the K_C of rock specimens. Therefore, the composite function based on exponents and logarithms was used to describe the variation trend between

K_C and $\dot{K}$ of ENDC sandstone specimens under static and dynamic fracture loading in this work, as shown in Figure 12. This function can well characterize the changing trend under static and dynamic loading. Based on the formula proposed in this paper, the data of other scholars [30, 32, 34, 57] are fitted, and it is found that the formula form can better represent the relationship between loading rate and fracture toughness.

4.2 | Relationship Between K_{IIC} and K_{IC}

From Figures 11 and 12, it appears that there might be a relationship between K_{IIC} and K_{IC} . Some data on K_{IIC} and K_{IC} for ENDB and ENDC specimens were collected and plotted in Figure 13, along with the data from this work. It can be observed that K_{IIC} and K_{IC} exhibit a strong linear relationship [23, 24, 35, 36, 58, 59]. This proportional relationship is significantly influenced by the loading form (ENDC or ENDB). Under static loading, the proportionality coefficient of ENDC specimens is greater than 1.00 and less than 1.00 for ENDB specimens. This indicates that the value of K_{IIC} for ENDC is greater than that of K_{IC} , whereas the value of K_{IIC} is smaller than that of K_{IC} for ENDB. Based on the collected data, the relationship between K_{IIC} and K_{IC} does not significantly depend on the rock type or origin but is strongly influenced by the loading form (ENDC or ENDB). Additionally, the loading state (dynamic or static) also has a great impact. Under dynamic loading, the relationship between K_{IIC} and K_{IC} changed for the ENDC

specimens in this work, in contrast to static loading, where the dynamic K_{IIC} was slightly smaller than the K_{IC} . Consequently, predicting the K_{IIC} of the same specimen type and loading mode may be possible using this proportional relationship.

4.3 | Fracture Surface Morphology Analysis

Besides mechanical properties and fracture toughness, the crack propagation path is often a focus of study for researchers. However, unlike mode I and mode II fracture loading, it is challenging to visually monitor crack propagation in mode III fracture loading using techniques such as the digital image correlation method [60] or high-speed imaging [61]. Drawing on previous research, the morphology of fracture surfaces can serve as an essential characteristic of rock fracture behavior.

4.3.1 | Static Test

After reaching the peak, the specimens experience failure and rupture. The typical post-failure fracture morphologies of the specimens are depicted in Figure 14. The fracture surfaces of the specimens under mode I fracture loading extend along the plane of the prefabricated cracks, whereas the mode III fracture-loaded specimens show an out-of-plane tearing failure. In terms of crack propagation, the crack under

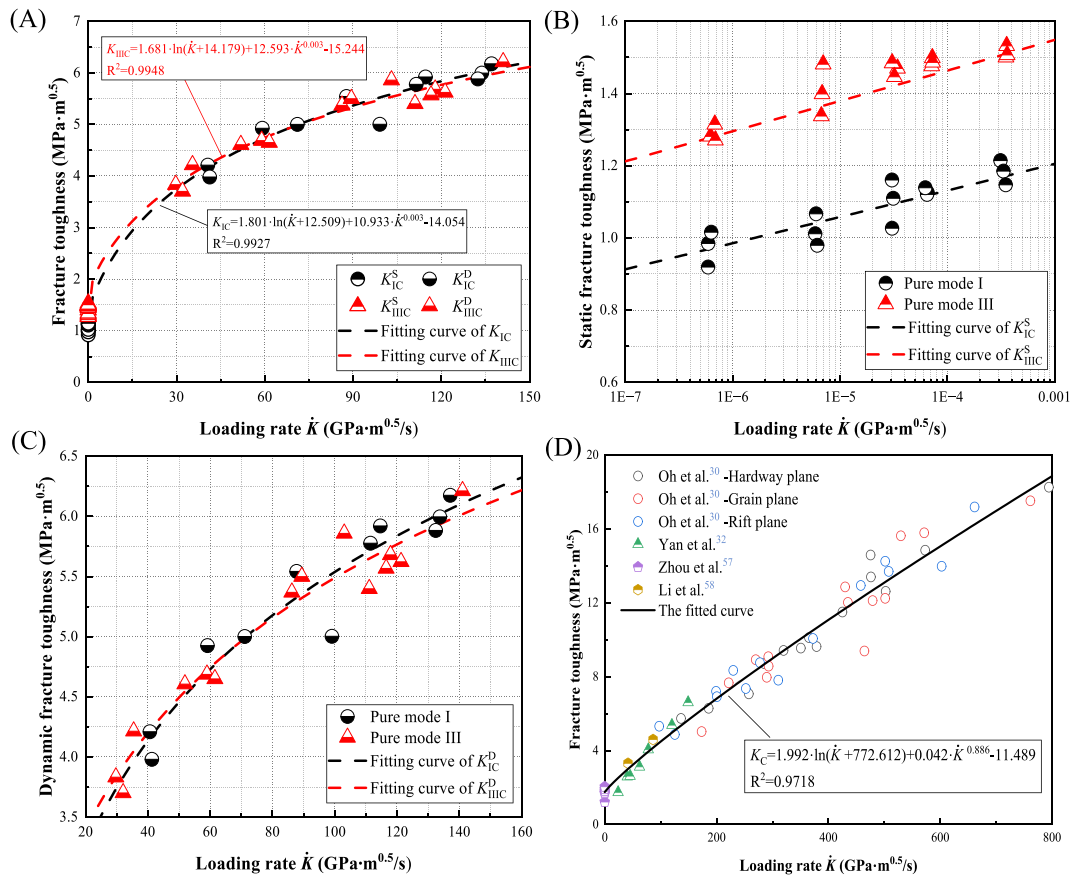


FIGURE 12 | Loading rate-dependent fracture toughness of ENDC sandstone sample: (A) global image, (B) static scale (locally enlarged image), (C) dynamic scale (locally enlarged image), and (D) other data validation. [Colour figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com/doi/10.1111/ife.14538)]

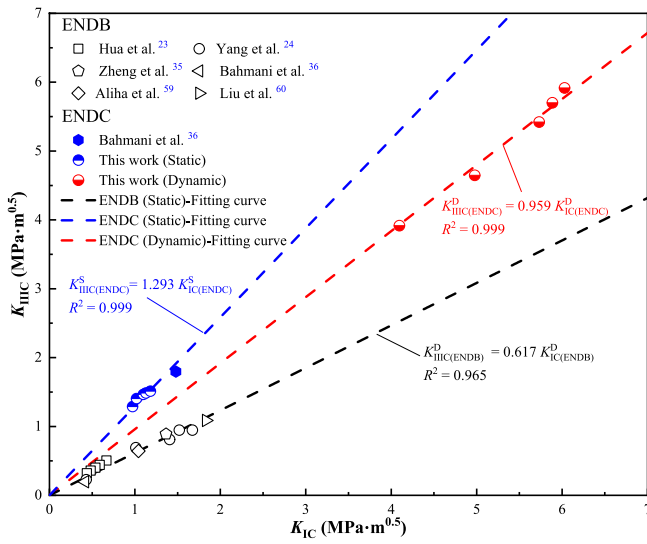


FIGURE 13 | Relationship between K_{IIC} and K_{IC} . [Colour figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com)]

mode III fracture loading first propagates approximately straight along the direction of the prefabricated notch from the center of the disc ($Z=0$) for a certain distance and then deflects toward the edge of the disc where the load is applied. It is worth noting that the length of this nearly straight propagation without deviation is influenced by the loading speed. On the side of the prefabricated notch, the length of this nearly straight propagation and the angle at which deviation occurs increase with higher loading speeds. This phenomenon is most prominent on the side where the prefabricated notch is located. This finding corresponds with the results obtained by Bahmani and Nematı [62], who pointed out that the extent of this phenomenon largely depends on the loading mode, mixing ratio, and the tensile-tearing deformations at different locations of the crack front. In this work, we further emphasize that not only the loading mode, but also the loading speed, even under the same loading mode, is an important influencing factor.

4.3.2 | Dynamic Test

In terms of the internal fracture surfaces of the specimens (as shown in Figure 14), the fracture surfaces of the specimens (i.e., the main propagation path of mode III cracks) subjected to mode III fracture loading show a significant inclination. However, from an intuitive perspective, the external fracture morphology of specimens subjected to mode III and mode I fracture loading shows a high degree of similarity. On the one hand, crack propagation under mode III and mode I fracture loading becomes more extensive with increasing impact pressure, resulting in a higher degree of specimen fragmentation. On the other hand, specimens under both loading conditions exhibit similar end failures on both loading sides. This end-face failure results from the instantaneous stress concentration that occurs when the bars impact the specimens.

To further investigate the difference in fracture behavior of ENDC sandstone specimens under dynamic and static fracture loading, in this work, the geometric morphology of the fracture

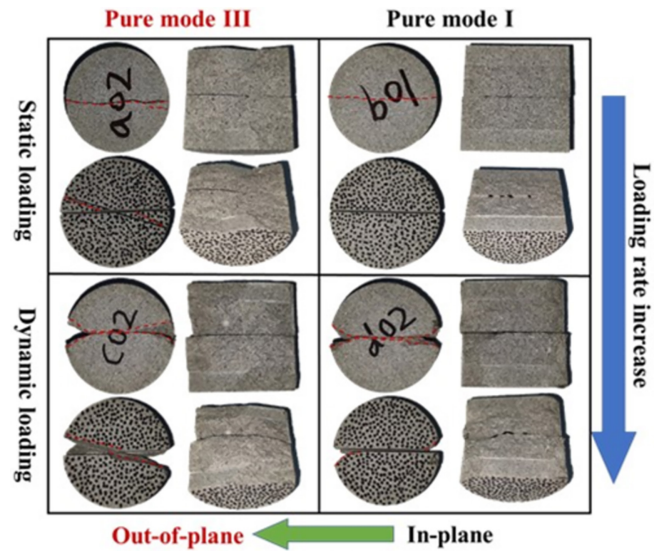


FIGURE 14 | Typical fracture morphology of fractured ENDC sandstone specimen. [Colour figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com)]

surfaces was quantitatively analyzed by using the joint roughness coefficient (JRC) and fractal dimension based on three-dimensional laser scanning technology.

The JRC [63, 64] is a crucial parameter for describing the characteristics of fracture surfaces and is used for evaluating crack propagation stability. The fracture surface of the specimen was first scanned using a three-dimensional laser scanning system, as shown in Figure 15A. Then, the fracture surface was reconstructed based on the scanned three-dimensional point cloud data using Rhino (a three-dimensional modeling software). This work distinguished from Shi et al. [25], who used the entire reconstructed surface as the region of interest (ROI). Considering that the main research crack only propagates on the non-prefabricated notch side of the specimen, the ROI is reset. Although cracks also appeared on the prefabricated notch side of the specimen (as can be observed in Figures 14), such cracking is caused by the stress concentration at both ends during loading and is not the main subject of study. To avoid the influence of these additional fractures, only the side without the prefabricated notch (50 mm in length along the X-direction and 15 mm in width along the Y-direction) was selected as the ROI, as shown in Figure 15A. After cropping the fracture surface using Rhino software, the cropped fracture surface was then reconstructed into a grid of 0.5×0.5 mm grid cells based on the MATLAB platform. Subsequently, the grid data were extracted along the Y-direction, resulting in 31 two-dimensional profiles of 50 mm length along the X-direction. The JRC was calculated using the empirical formula proposed by Tse and Cruden (as seen in Equation 6) [65].

$$JRC = 32.2 + 32.47 \cdot \log Z_2 \quad (6a)$$

$$Z_2 = \left[\frac{1}{M(\Delta y)^2} \sum_{i=1}^M (z_{i+1} - z_i)^2 \right]^{\frac{1}{2}} \quad (6b)$$

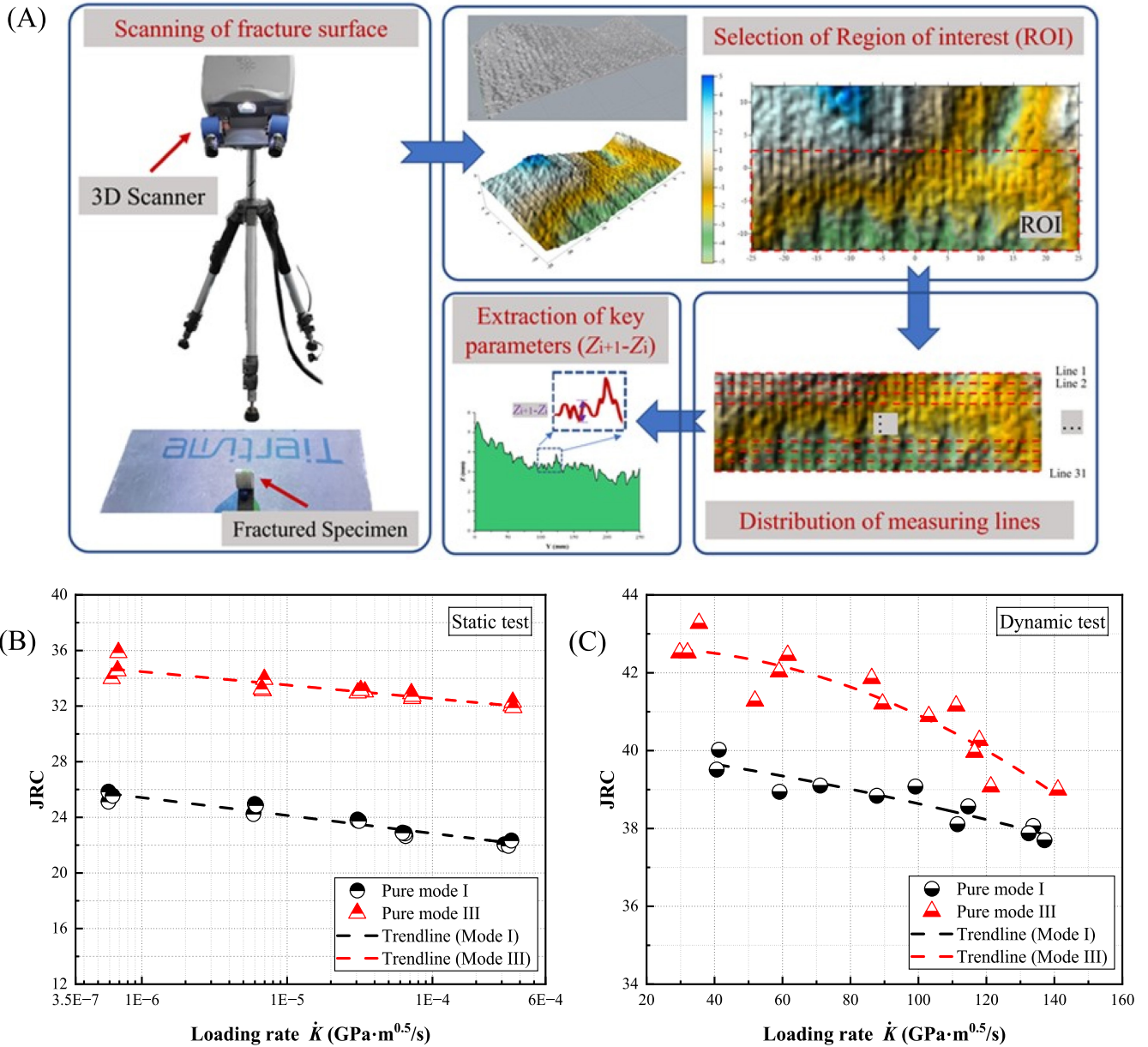


FIGURE 15 | Relationship between JRC of fracture surfaces and loading rate of the ENDC sandstone specimen. (A) Schematic diagram of calculation method of JRC of fracture surface. (B) Static test. (C) Dynamic test. [Colour figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com/doi/10.1111/rfm.14538)]

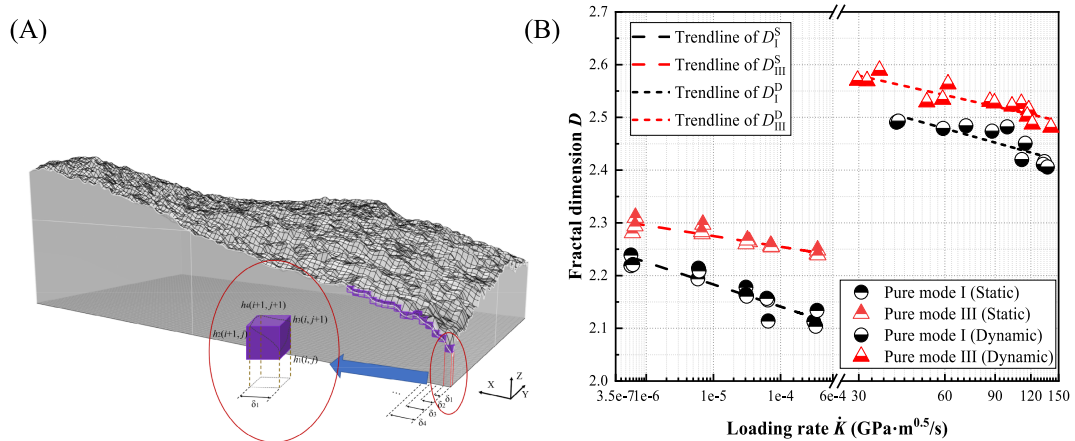


FIGURE 16 | Fractal dimension evaluation of fracture surface. (A) Schematic diagram of calculation method of fractal dimension of fracture surface. (B) Relationship between fractal dimension of fracture surfaces and loading rate of the ENDC sandstone specimen. [Colour figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com/doi/10.1111/rfm.14538)]

where Z_2 is the root mean square, Δy is the sampling interval, M is the amount of interval, and $z_{i+1}-z_i$ is the height difference in the Z -direction between two adjacent sampling points, taking one profile as an example in Figure 15A.

Figure 15B,C shows how the JRC varies with the K . The fracture surface of the mode III fracture-loaded specimens is rougher under both dynamic and static loading. It may be because the propagation of the out-of-plane tearing crack front is affected by more faces and edges of the irregular mineral grains, as well as having a higher degree of freedom out of plane. In static loading tests, the JRC of the fracture surface under both loading modes decreases as the K increases. In dynamic loading tests, a similar trend is observed where the JRC decreases with higher K . This observation may be explained by the crack propagation mode, that is, cracks tend to propagate in the form of transgranular cracks at higher K , which makes the fracture surfaces smoother. The same phenomenon was also found in the study of Aghababaei, Behnia, and Aliha [66]. However, because the JRC is also affected by the grain size of rock to some extent, the JRC of different rock samples lacks comparative significance. It should be noted that for the dynamically loaded specimens, the fracture surfaces are the main fracture surface canned after assembling the fractured fragments to eliminate the effect of additional fracture surfaces as much as possible.

In addition to using the JRC to evaluate the geometric morphology of fracture surfaces, the fractal dimension (D) is also widely applied in studying crack propagation. To further quantify the complexity of fracture surface morphology, the three-dimensional box dimension method [25, 26] was employed to calculate the characteristics of the fracture surfaces, as shown in Figure 16A. The fracture surface was divided into square grids of side length δ in the XOY plane, and the fracture surface of the specimen was covered with cubes of side length δ . The total number of cubes $N(\delta)$ was then counted. Based on Equation (7), the slope of $\ln N(\delta) \sim \ln \delta$, that is, the D , was fitted using the least-squares method for different values of δ [25].

$$D = -\delta \lim_{\delta \rightarrow 0} \frac{\ln N(\delta)}{\ln \delta} \quad (7)$$

As shown in Figure 16B, the D of the fracture surfaces does not exhibit a simple decrease with increasing loading rate. Within the same loading range (either static or dynamic loading), the D decreases with an increasing K . However, the overall trend does not conform to this law. For quasistatic loading, the relationship between the D of the fracture surface and the K is still unclear, and further work is needed to investigate this relationship.

5 | Conclusions

In this work, different loading states were applied to ENDC sandstone specimens, including static mode III fracture loading with various loading speeds and dynamic mode III fracture loading driven by different impact pressures. The strength and fracture characteristics under static and dynamic loading were analyzed, and the mode III fracture characteristics of ENDC sandstone

specimens loaded at different “extents” of loading under static and dynamic loading were compared in a normalized manner using the loading rate index. Furthermore, the proportionality between mode III and mode I fracture toughness of ENDC specimens was discussed. The main conclusions are as follows:

1. Under static mode III fracture loading, the P_0 and K_C of ENDC sandstone specimens increase with increasing v . In addition, the mode III fracture-loaded specimens have higher static K_C compared to mode I fracture-loaded specimens, which is 1.28–1.38 times higher than that of K_{IC} . Under dynamic mode III fracture loading, the P_0 and K_C of ENDC sandstone specimens increase with the increase of p . Unlike static loading, the mode I fracture-loaded specimens exhibit higher dynamic fracture toughness.
2. The K increases with increasing v (under the static test) and impact pressure (under the dynamic test). The K_{IIC} and K_{IC} of ENDC sandstone specimens increase with an increasing of K . However, the overall difference between K_{IIC} and K_{IC} is slight. Notably, the size relationship between K_{IIC} and K_{IC} values under static and dynamic loading undergoes a shift in size. Furthermore, this relationship between K_{IIC} and K_{IC} values is not significantly related to the rock type or origin but is significantly affected by the loading form (ENDC or ENDB).
3. Under both dynamic and static loading, the joint roughness coefficient and fractal dimension of the fracture surface decrease with increasing K . The crack propagation mode primarily influences this change. However, the relationship between the geometric characteristics of the fracture surface and the K under quasistatic loading remains unclear and requires further investigation.

Nomenclature

K	the loading rate
K	the stress intensity factor
K_I, K_{III}	the stress intensity factor of mode I (III) fracture
K_C	the fracture toughness
K_{IC}, K_{IIC}	the fracture toughness of mode I (III) fracture
K_{IC}^D, K_{IIC}^D	the dynamic fracture toughness of mode I (III) fracture
K_{IC}^S, K_{IIC}^S	the static fracture toughness of mode I (III) fracture
Y_I, Y_{III}	the geometric factor of mode I (III) fracture
R	the radius of ENDC specimen
B	the thickness of ENDC specimen
a	the length of the notch
β	the projected angle between the prefabricated notch and the loading force
P, P_0	the load (peak load) applied to the ENDC specimen
$P_1(t), P_2(t)$	the time-dependent dynamic force at the end of the specimen on the specimen on the incident (transmitted) bar
$\varepsilon_I(t), \varepsilon_R(t), \varepsilon_T(t)$	the time-dependent strain in the incident (reflected, transmitted) bar
A	the cross-sectional area of the bar

E	the elastic modulus of the bar
v	the loading speed
p	the impact pressure
Δy	the sampling interval
M	the amount of interval
$z_{i+1} - z_i$	the height difference in the Z-direction between adjacent sampling points
D	the fractal dimension

Author Contributions

Xiaofeng Qin: conceptualization, writing – original draft, methodology, project administration, investigation, formal analysis, visualization, data curation. **Haijian Su:** conceptualization, writing – review and editing, methodology, project administration, funding acquisition. **Liyuan Yu:** methodology, resources, supervision, funding acquisition. **Hao Wang:** investigation, data curation. **Ying Jiang:** data curation, resources. **Thi Nhan Pham:** data curation, validation.

Acknowledgments

This study is financed by the National Natural Science Foundation of China (Nos. 52274100, 52179118, and 42077240).

Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

Data will be made available on request.

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