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GNSS-based Height Determination in Vietnam: an easy-to-use approach utilizing GNSS/leveling corrector surface

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Abstract. Accurate transformation of GNSS-derived ellipsoidal heights to physical heights remains a fundamental requirement for geodetic and engineering applications. In Vietnam, conventional approaches rely on dense gravimetric observations and the development of local quasigeoid models, which are costly and time-consuming to maintain. This study presents an alternative easy-to-use and practical framework for GNSS-based leveling height determination by integrating global geopotential models (GGMs) with existing GNSS/leveling data through a corrector surface approach. Several state-of-the-art high-degree GGMs, including EGM2008, EIGEN-6C4, SGG-UGM-2, and XGM2019e, are evaluated using 816 co-located GNSS/leveling points distributed across the Vietnamese mainland. The results demonstrate that the XGM2019e model provides the best overall performance, particularly at high spherical harmonic degrees. Systematic biases and long-wavelength discrepancies between the global model and the national height system are mitigated using a four-parameter deterministic Helmert transformation, while remaining medium- and short-wavelength residuals are modeled by least-squares collocation. Validation against GNSS/leveling data shows that the proposed corrector surface reduces the standard deviation of height anomaly differences from ± 0.153 to ± 0.083 m. A gridded corrector surface at 5-arcminute resolution is subsequently generated, enabling efficient transformation of GNSS ellipsoidal heights into leveling heights consistent with the Vietnam national height system. The proposed methodology offers a cost-effective, scalable, and easily updatable alternative to local gravimetric quasigeoid models, particularly suitable for regions with limited gravity data coverage.

Keywords: GNSS, leveling, geoid, corrector surface, Vietnam

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Определение высот на основе ГНСС во Вьетнаме: простой в использовании подход с применением корректирующей поверхности ГНСС/нивелирования

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Аннотация. Точная трансформация эллипсоидальных высот, полученных с помощью ГНСС, в физические высоты остаётся фундаментальным требованием в геодезических и инженерных приложениях. Во Вьетнаме традиционные подходы опираются на плотные гравиметрические наблюдения и создание локальных квазигеоидов, что является дорогостоящим и трудоёмким процессом. В данном исследовании предлагается альтернативная, простая в использовании и практичная методика определения высот на основе ГНСС, основанная на интеграции глобальных моделей гравитационного поля (GGM) с существующими данными ГНСС/нивелирования посредством подхода корректирующей поверхности. В работе оцениваются несколько современных высокостепенных глобальных моделей, включая EGM2008, EIGEN-6C4, SGG-UGM-2 и XGM2019e, с использованием 816 совмещённых пунктов ГНСС/нивелирования, равномерно распределённых по территории Вьетнама. Результаты показывают, что модель XGM2019e обеспечивает наилучшее качество по сравнению с другими моделями, особенно при высоких степенях сферических гармоник. Систематические смещения и долговолновые расхождения между глобальной моделью и национальной системой высот устраняются с помощью четырёхпараметрического детерминированного преобразования Гельмерта, тогда как средне- и коротковолновые остатки моделируются методом коллокации наименьших квадратов. Проверка результатов по данным ГНСС/нивелирования показывает, что предложенная корректирующая поверхность уменьшает среднеквадратическое отклонение различий аномалий высот с ± 0.153 м до ± 0.083 м. Затем формируется корректирующая поверхность с разрешением 5 угловых минут, что позволяет эффективно преобразовывать эллипсоидальные высоты ГНСС в нивелирные высоты, согласованные с национальной системой высот Вьетнама. Предлагаемая методика представляет собой экономичную, масштабируемую и легко обновляемую альтернативу локальным гравиметрическим квазигеоидам, особенно в районах с ограниченным покрытием гравиметрическими данными.

Ключевые слова: ГНСС, выравнивание, геоид, поверхность корректора, Вьетнам

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Introduction

The heights of points on the Earth surface can be observed by the traditional spirit leveling, which is one of the most accurate methods. In this way, the

height differences between points are measured using levels and leveling staffs or rods. The accuracy of the leveled heights depends on the distances between observed points, the number of station setups, and

the observation time duration [1]. In a more strict sense of physical geodesy, geodetic leveling can be adopted combining spirit leveling with gravimetric observations by which a gravity potential difference can be computed [2].

However, spirit or geodetic leveling is time-consuming and cost inefficient. Spirit leveling is beneficial over short distances due to its small random errors; However, over long distances, systematic errors dominate random errors [2], leading to lower accuracy compared to the GNSS-based method. To reduce the influence of systematic errors in leveling, a network of leveling loops combined with multiple coastal tide gauges are usually adopted. This approach was carried out to establish national leveling networks in many countries, which are split into different orders, such as first-, second-, and third-order [1, 3]. Additionally, strict leveling observation standards are used in high-order networks, such as limiting the difference in distances from the station to the forward and backward leveling staffs.

With the advent of GNSS systems, the positioning of points on the Earth surface is more convenient, which provides the positions in both horizontal and vertical directions. In using GNSS, the geometrical height of a point with respect to a reference ellipsoid, i.e., ellipsoidal height, is determined. This can be observed directly between two GNSS receiver locations without measuring height increments along a leveling route. However, physical height, e.g., orthometric or normal height with respect to geoid or quasigeoid, is still needed in practical applications. In this case, the separation between ellipsoid and geoid/quasigeoid is needed, which are known as geoid undulation or height anomaly.

Recently, many geospatial data incorporating ellipsoidal heights are utilized, e.g., airborne or spaceborne light detection and ranging (LiDAR) [4], satellite altimetry [5], Interferometric Synthetic Aperture Radar (InSAR) [6]. In these data, the ellipsoidal heights of an airborne or spaceborne platform, while moving on its trajectory or orbit, are determined relying on onboard GNSS receivers. These are subsequently subtracted by the platform's altitudes or ranges measured with respect to the Earth surface to derive the ellipsoidal heights of the Earth. This geometric ellipsoidal height needs to be transformed to physical orthometric or normal height for practical use.

Geoid or quasigeoid models are useful in determining GNSS-based heights by transforming

ellipsoid heights to physical heights. Multiple Global Geopotential Models (GGMs) were made by combining data of different types, e.g., satellite altimetry, surface and airborne/satellite gravimetry, GNSS, leveling. In using these models, geoid undulations or height anomalies can be computed for any point with known coordinates latitude, longitude by either interpolation from predefined grids or harmonic coefficients with fully associated Legendre function. These GGMs best fit on a global scale [3], and are good in areas where available data were used in constructing the models [7–9].

Alternatively, several local geoid models were also generated, most of them relying on local gravimetry data by either the least-squares Collocation (LSC) method or the Fast Fourier Transform (FFT) method relying on Stokes' integral [e.g., 10], which best fit on a local or regional scale. Recently, there has been a demand for unifying the height system on the global scale via the so-called International Height Reference System (IHRs), which is based on geopotential numbers rather than conventional height [9]. Similarly, a related quantity of geopotential value and its variations in time were also studied on a national scale utilizing the geodetic boundary value problem in line with global gravity field model and surface gravity data [e.g., 3].

The approaches to derive geoid models, via geoid undulation or geopotential numbers mentioned above, are rigorous but require gravimetric observations and sophisticated physical and mathematical models. This paper aims to apply an easy-to-use approach to derive GNSS-based normal heights in Vietnam as an alternative to the use of local gravimetric or GNSS/leveling quasigeoid models. The objectives of this paper are three-fold, including i) choosing the global high degree and order (d/o) GGM model closest to the Vietnam normal height system, ii) creating a four-parameter model reflecting geoid/GNSS/leveling differences to account for the biases and tilts between global and local GNSS/leveling geoid models, iii) creating a corrector surface using the statistical interpolation method Collocation applied to the remaining GGM/GNSS/leveling residuals.

This approach is beneficial in that it not only provides an easy-to-use way to derive GNSS-based normal heights, but is also easy to be updated once a new global GGM model is released with improvements, e.g., improved data resolution and amount, more advanced data processing approaches and standards [9], or additional local GNSS/leveling

data are added. A new GGM model could also be released to reflect the change in potential in time of a global geoid model due to mean sea level rise and the mass redistribution [3]. Even though GNSS/leveling data have been used in validating several global GGMs in Vietnam [e.g., 7], this only compared those models to indicate how good they are by comparing to leveling heights. However, a model still needs to be developed for conveniently determining normal heights in engineering applications. The results in this study are validated by comparing the corrector surface-based heights with the leveling derived normal heights.

Datasets
Global Geopotential Models

Different GGMs are used in this study to select the best fit model in Vietnam. These tested models include the most widely used Earth Gravitational Model 2008 (EGM2008) [11], and the most recent models of European Improved Gravity model of the Earth by New techniques (EIGEN-6C4) [12], SGG-Unified Gravity Model second version (SGG-UGM-2) [13], and Experimental Gravity Field Model 2019 (XGM2019e) [14] (table 1). In this study, we use the fully normalized spherical harmonic coefficients of the Earth’s gravitational potential provided for each model to estimate height anomalies with respect to the World Geodetic System 1984 (WGS84) in the tide free system [10].

EGM2008 is a global model developed and published by the National Geospatial-Intelligence Agency (NGA, U.S.) in 2008. It is provided with fully normalized spherical harmonic coefficients up to d/o 2190 and offers grids of ~5 arcminutes. EGM2008 was developed based on combined satellite altimetry, ground and satellite data [11]. EIGEN-6C4 is another high degree model developed and published in 2014 by the GeoForschungsZentrum (GFZ) German Research Center for Geosciences and the French Space Agency (CNES). Like EGM2008, EIGEN-6C4 was provided with spherical coefficients up to d/o 2190 corresponding to a spatial resolution of ~5 arcminutes. EIGEN-6C4 was developed using combined data of satellite altimetry, satellite gravimetry (Gravity Field and Steady-State Ocean Circulation Explorer – GOCE, Gravity Recovery and Climate Experiment – GRACE), satellite laser ranging (SLR) (Laser Geodynamics Satellites – LAGEOS), and terrestrial data [12].

SGG-UGM-2 is a high degree gravitational model published in 2020 by the School of Geodesy and Geomatics (SGG) (Wuhan University, China) following its predecessor SGG-UGM-1. It was completed to d/o 2190. Like EIGEN-6C4, SGG-UGM-2 was developed by using combined data of satellite altimetry, satellite gravimetry (GRACE, GOCE), SLR, and terrestrial data [13]. Notably, both EIGEN-6C4 and SGG-UGM-2 were based on EGM2008 as a reference or for generating complementary data. XGM2019 is a combined global gravitational model at ultra-high degree (spheroidal harmonics up to d/o 5399 corresponding to a spatial resolution of 2 arcminutes) developed by the GFZ and the Technical University of Munich (TUM), which was published in 2019. It was developed based on the satellite model GOCO06s, terrestrial data supported by NGA, topographic gravity data over the land and satellite altimetry derived gravity anomalies over the oceans [14]. Unlike EIGEN-6C4 or SGG-UGM-2, XGM2019 was not based on the commonly used model EGM2008. Although the error of XGM2019 was reported to be larger than that of other existing high-degree models over the well-surveyed areas because of the sole use of topographic forward modeling, it is considered to be globally more homogeneous over the global scale and independent of existing models [14]. XGM2019 is provided with three products with spherical harmonics up to d/o 760, 2190, and 5540 of which the two products at the highest d/o XGM2019e2159 and XGM2019e5540 are used in this study (table 1).

Table 1. Global Geopotential Models tested in this study.
GFZ – GeoForschungsZentrum, NGA – the National Geospatial-Intelligence Agency, TUM – the Technical University of Munich

Model	Published year	Published agency	Maximum degree
EGM2008	2008	NGA (U.S.)	2190
EIGEN-6C4	2014	GFZ Potsdam (Germany)	2190
SGG-UGM-2	2020	Wuhan University (China)	2190
XGM2019e2159	2019	TUM (Germany)	2190
XGM2019e5540	2019	TUM (Germany)	5540

GNSS/leveling Data

In this study, we use 816 co-located GNSS/leveling points distributed relatively regularly in the Vietnam's mainland to generate a corrector surface (fig. 1). They belong to the Vietnam national leveling networks with 235 first-order, 197 second-order, and 384 third-order. The leveling observations were measured by high precision spirit levels during the years 2001–2003. They were measured by the double-run method, i.e., forward and backward runs were made in each leveling route, and the same distance to foresight and backsight staff at each station setup was designed. The closed loop or closed link misclosure limits applied to the Vietnam first-, second-, and third-order leveling networks are $\pm 2\sqrt{L}(\pm 3\sqrt{L})$, $\pm 4\sqrt{L}(\pm 5\sqrt{L})$, and $\pm 10\sqrt{L}(\pm 12\sqrt{L})$, respectively, for flat (mountainous) areas, with L being the total distance of the loop in km [7, 15]. The networks were adjusted using the 1950–2005 local mean sea level at the Hon Dau tide gauge station (fig. 1), by which normal heights (H^*) were estimated [3].

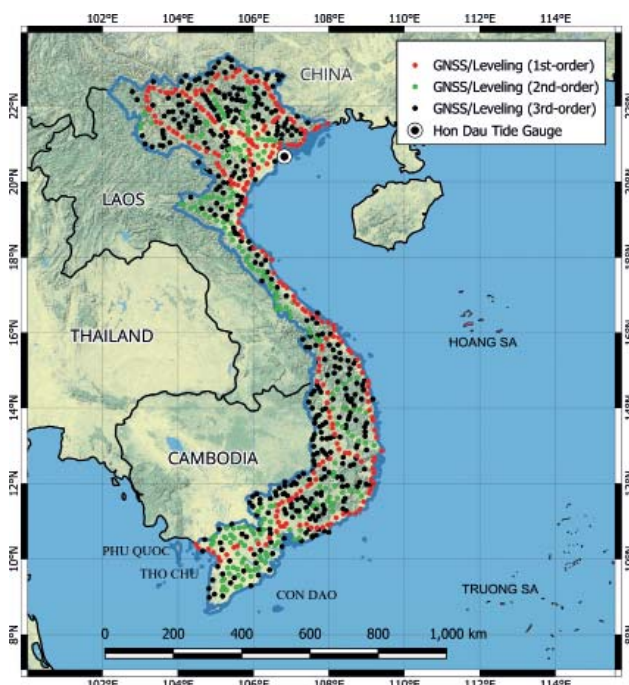


Fig. 1. Co-located GNSS/leveling points forming first-order (red), second-order (green), and third-order (black) leveling networks

GNSS observations were then made over the co-located points by the Vietnam Department of Surveying, Mapping and Geographic Information during the years 2009–2010, using dual-frequency GNSS receivers with the static relative method and a minimum observation duration of 6 hours per session

[3, 7]. The GNSS observations were processed and adjusted by the Bernese software package [16], by which ellipsoidal heights (h) with respect to WGS84 were derived. Finally, height anomalies ($\zeta^{GNSS/lev}$) were computed for the entire 816 co-located points [10]:

$$\zeta^{GNSS/lev} = h - H^*. \quad (1)$$

The maximum, minimum, mean, and standard deviation of computed height anomalies are 5.601 m, –33.168 m, –14.685 m, and ± 12.298 m, respectively.

Methods

This study applies the workflow in fig. 2 to generate a GGM/GNSS/leveling corrector surface to derive GNSS-based normal heights in Vietnam. With the GNSS/leveling points shown in Section 2.2, GNSS/leveling height anomalies ($\zeta^{GNSS/lev}$) are computed using Equation (1).

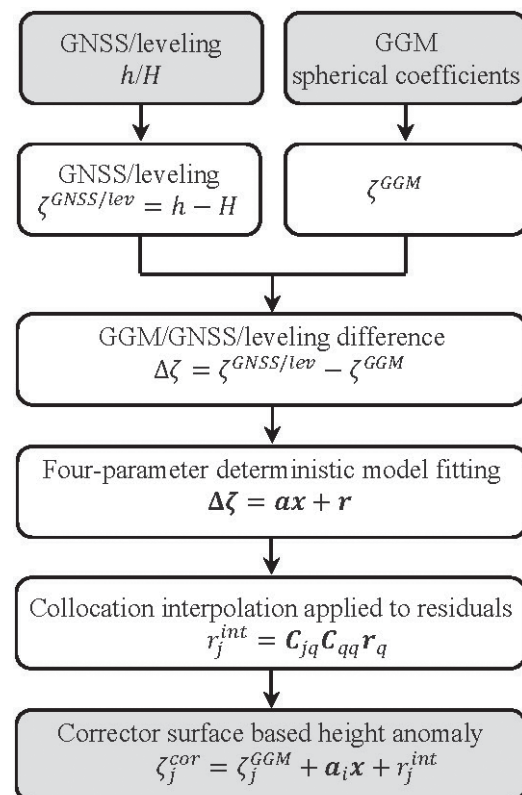


Fig. 2. Workflow of corrector surface generation

At the same time, GGM-based height anomalies (ζ^{GGM}) are computed using spherical harmonic coefficients provided with the tested models shown in table 1 (Section 3.1). The GGM/GNSS/leveling differences are then computed for all co-located points, which are the difference between $\zeta^{GNSS/lev}$ and ζ^{GGM} . These differences are then modeled using the four-parameter Helmert mathematical model

(Section 3.2). The residuals of the least-squares fit model are subsequently interpolated using the statistical collocation method (Section 3.3). Finally, at a point without leveled height, GNSS can be applied in line with the generated corrector surface to derive corrector surface based normal height (H^{cor}).

Computation of GGM-based Height Anomalies

With the fully-normalized spherical harmonic coefficients provided for each GGM, the height anomaly (ζ^{GGM}) of a point with spherical coordinates latitude (φ) longitude (λ), and geocentric radius (ρ) can be computed from the disturbing potential [17–19]:

$$\zeta^{GGM} = \frac{GM}{\gamma\rho} \left[\sum_{n=2}^{N_{\max}} \left(\frac{a}{\rho} \right)^n \sum_{m=0}^n \left(\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda \right) \bar{P}_{nm}(\sin \varphi) \right], \quad (2)$$

where GM is the product of the Newtonian gravitational constant and the mass of the geopotential model, γ is the normal gravity at the estimated point with an ellipsoidal height h , a is the equatorial radius of the geopotential model, $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are the fully-normalized spherical harmonic coefficients of degree n and order m , $N_{\max}$ is the maximum degree of the geopotential model, $\bar{P}_{nm}$ is the fully normalized associated Legendre's function (ALF) that is computed by the recursive method [17, 18, 20, 21]:

$$\begin{aligned} \bar{P}_{nm}(\sin \varphi) &= a_{nm} \sin \varphi \bar{P}_{n-1,m}(\sin \varphi) - b_{nm} \bar{P}_{n-2,m}(\sin \varphi), \\ &\quad n > m, \\ \bar{P}_{nm}(\sin \varphi) &= c_{nm} \cos \varphi \bar{P}_{m-1,m-1}(\sin \varphi), \end{aligned} \quad (3)$$

here

$$\begin{aligned} a_{n,m} &= \sqrt{\frac{(2n-1)(2n+1)}{(n-m)(n+m)}}, \\ b_{n,m} &= \sqrt{\frac{(2n+1)(n+m-1)(n-m-1)}{(n-m)(n+m)(2n-3)}}, \\ c_{m,m} &= \sqrt{\frac{(2m+1)}{2m}} \end{aligned} \quad (4)$$

and the initial values of the fully normalized ALF are [17, 18, 20]:

$$\begin{aligned} \bar{P}_{00}(\sin \varphi) &= 1, \quad \bar{P}_{10}(\sin \varphi) = \sqrt{3} \sin \varphi, \\ \bar{P}_{11}(\sin \varphi) &= \sqrt{3} \cos \varphi. \end{aligned} \quad (5)$$

Here, height anomalies are computed between WGS84 ellipsoid and GGMs with the scaling factors $GM = 3,986,004.415 \cdot 10^8 \text{ m}^3/\text{s}^2$, $a = 6,378,137 \text{ m}$.

The zonal coefficients $\bar{C}_{20}$, $\bar{C}_{40}$, and $\bar{C}_{60}$ are corrected for the differences between actual coefficients and those of the reference equipotential ellipsoid following Rapp [22], and the normal gravity γ is computed following Moritz [23].

Deterministic Transformation Model Fitting

Once the GNSS/leveling height anomalies $\zeta^{GNSS/lev}$ and GGM-based height anomalies ζ^{GGM} are computed for all co-located points applying Equations (1) and (2), the differences between them can be generated:

$$\Delta\zeta^{GNSS/lev-\zeta^{GGM}}. \quad (6)$$

The differences $\Delta\zeta$ are theoretically zero if all components GNSS/leveling observations and the GGM model were made perfectly. In practice, different noise and errors existing in the GNSS/leveling observations, datum inconsistencies between the two systems of GNSS/leveling and GGM models, geodynamic effects, and theoretical approximations, result in nonzero misclosures $\Delta\zeta \neq 0$ [24, 25].

In order to deal with the datum inconsistencies and systematic effects, the nonzero misclosures can then be modeled using a deterministic transformation model for fitting the differences between GNSS/leveling and GGM-based height anomalies for the entire network of co-located points [24]:

$$\Delta\zeta = ax + r, \quad (7)$$

where $\Delta\zeta$ is the vector of GGM/GNSS/leveling differences of n co-located points $[\Delta\zeta_1, \Delta\zeta_2, \dots, \Delta\zeta_n]^T$, x is the vector of m unknown parameters $[x_1, x_2, \dots, x_m]^T$, r is the vector of n residuals $[r_1, r_2, \dots, r_n]^T$, a is the array of known coefficients of the size $n \times m$:

$$a = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{bmatrix}. \quad (8)$$

Different deterministic models have been applied in the literature of which the most widely used four-parameter ($m = 4$) Helmert similarity transformation model is applied in this study [7, 24–26]:

$$\Delta z = a_i x + r_i = x + \cos \phi_i \cos \lambda_i x_2 + \cos \phi_i \sin \lambda_i x_3 + \sin \phi_i x_4 + r_i, \quad (9)$$

where ϕ_i and λ_i are the geodetic latitude and longitude of the considered point i , $x_j (j = 1, \dots, 4)$, is the unknown coefficients, r_i is the residual. The unknown vector x is solved by applying the least-squares principle to minimize the sum of the squared residuals r^2 .

Least-squares Collocation Interpolation

After applying the deterministic transformation model in Section 3.2, a vector of 816 residuals (r_q), incorporating random errors from GNSS/leveling observations and the GGM, is derived for the co-located points. These residuals are subsequently modeled by the statistical least-squares collocation method from which the residual of a new point $j(r_j^{int})$ can be computed [27]:

r_j^{int} = C_{jq} C_{qq}^{-1} v_q = [C_{j1} C_{j2} \cdots C_{jq}] \times [C_{11} C_{12} \cdots C_{1q}; C_{21} C_{22} \cdots C_{2q}; \vdots \vdots \ddots \vdots; C_{q1} C_{q2} \cdots C_{qq}]^{-1} [r_1; r_2; \vdots; r_q], (10)

where q is the number of co-located GGM/GNSS/leveling points that are used (sample points), C_{jq} is the vector of covariances between the interpolated point j and q co-located points, C_{qq} is the array of the $q \times q$ size of covariances between co-located sample points, r_q is the vector of q residuals derived after fitting the deterministic transformation model.

Empirical covariances are first computed relying on q residuals of the co-located points then a covariance model is fit to derive the covariance parameters. In this study, we use a simple collocation function [28]:

C = C_0 e^{-d/L}, (11)

where C_0 is the function variance, d is the distance between points, L is the characteristic length. The function parameters C_0 and L are estimated using the least-squares method.

Results and Discussions

Selection of Best-fit GGM

In this section, high degree GGMs shown in table 1 are validated relying on GNSS/leveling height anomalies to select the best-fit model over the Vietnam mainland for use in subsequent steps shown in fig. 2. GGM-based anomalies are first computed for each model following Section 3.1, which are then compared with GNSS/leveling height anomalies to derive the GGM/GNSS/leveling differences $\Delta\zeta$. The descriptive statistical metrics of the differences, including the mean, standard deviation, maximum, and minimum values, are shown in table 2. In comparison between models at maximum degrees used, the EGM2008 model shows the worse standard deviation of ± 0.296 m, while the EIGEN-6C4 and SGG-UGM-2 models are of very close

standard deviation values of ± 0.196 and ± 0.192 m. The XGM2019e model with different products has the best metrics with the standard deviation values of ± 0.161 m and ± 0.156 corresponding to the maximum degrees of 2190 and 5540 used. These results confirm what have been concluded in [7].

Table 2. Comparison of GNSS/leveling and GGM height anomalies. The GGMs are listed in the descending order of standard deviation. Unit: m

Model	Max degree	Mean	Std Dev	Max	Min
EGM2008	2190	0.365	± 0.296	1.220	-0.563
EIGEN-6C4	2190	0.451	± 0.196	1.006	-0.591
SGG-UGM-2	2190	0.413	± 0.192	0.968	-0.636
XGM2019e2159	2190	0.420	± 0.161	1.039	-0.249
XGM2019e5540	5540	0.431	± 0.156	0.945	-0.110

We recall here that, except for the XGM2019e5540 product, all other models/products are provided with the same degree up to 2190. Therefore, we then validate those models at different d/o used to see how the standard deviation of GGM/GNSS/leveling differences progresses with the increase of d/o used. We test d/o from 5 to 2190 with a step of 5 with the results shown in fig. 3.

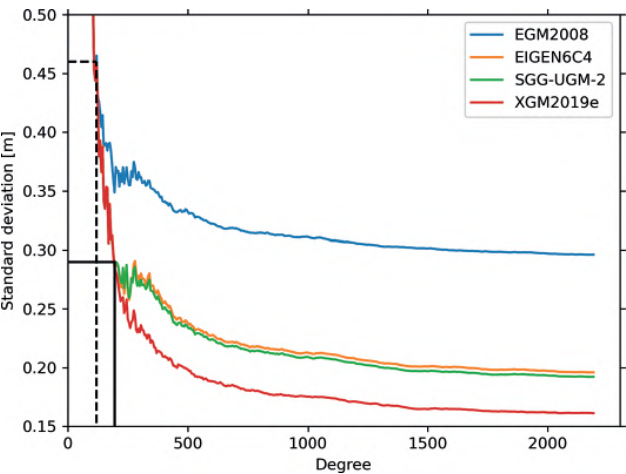


Fig. 3. Comparison of GGMs in the standard deviations at different d/o. The dashed black lines indicate the d/o of 120 where all four models are of the same standard deviation of $\sim \pm 0.46$ m. The solid black lines indicate the d/o 195 where the models EIGEN-6C4, SGG-UGM-2, and XGM2019 have similar standard deviation of $\sim \pm 0.29$ m, which are significantly better than the model EGM2008

The results show that at d/o up to 120 (black dashed line in fig. 3), all four tested models are of a similar standard deviation of $\sim \pm 0.46$ m. After

Table 3. Statistical summary of components used to validate the corrector surface model at co-located points. The GGM-based height anomalies are computed relying on the best fit model XGM2019e5540. Unit: m

Row	Component	Mean	Std Dev	Max	Min
2 nd	GGM/GNSS/leveling difference $\Delta\zeta$ (co-located points)	0.432	± 0.153	0.890	0.024
3 rd	$\Delta\zeta^{model}$ model-fitting (co-located points)	0.431	± 0.043	0.476	0.317
4 th	GGM/GNSS/leveling residual r	0.000	± 0.150	0.493	-0.574
5 th	Collocation interpolated residual r^{int} (grid points)	-0.004	± 0.135	0.466	-0.546

this d/o and up to d/o 195, the standard deviations of the three models EIGEN-6C4, SGG-UGM-2, and XGM2019e significantly reduce and are much better than that of EGM2008. Up to d/o 195, the standard deviation of those three models is $\sim \pm 0.29$ m, while that of EGM2008 is $\sim \pm 0.35$ m. After d/o 195 and up to the maximum d/o 2019, the model XGM2019e is superior to the other models. Therefore, the model XGM2019e5540 will be adopted hereinafter for further steps shown in fig. 2.

Corrector Surface Model Validation

To assess the corrector surface model's performance, we compare height anomalies estimated from the corrector surface model with GNSS/leveling height anomalies at co-located points (see table 3). To this end, GGM-based height anomalies are calculated using the best-fit XGM2019e model as shown in table 2, followed by computing differences between GGM and GNSS/leveling height anomalies (table 3, 2nd row). A four-parameter deterministic model is fitted to calculate both model-based height anomaly differences $\Delta\zeta^{model}$ (table 3, 3rd row) and residuals r (table 3, 4th row).

In comparison between the 2nd and 3rd rows of table 5, it can be seen that the distribution of GGM/GNSS/leveling differences is more dispersed than the modeled differences indicated by a larger standard deviation and range (i.e., the difference between the maximum and minimum values); specifically, the standard deviation of the two datasets are ± 0.153 m and ± 0.043 m, respectively, and their ranges are 0.866 m and 0.159 m, respectively. This indicates that potential datum inconsistencies between the two systems and systematic errors and noise in the GNSS/leveling and GGM have been modeled by the four-parameter fitting, but the random errors still exist. These inconsistencies and systematic errors may contain long/medium wavelength errors in the GGM, biases and trends in GNSS/leveling height anomalies, and systematic errors accumulated in the leveling data, particularly with points located at long distances from the Vietnam's local vertical datum at the Hon Dau tide gauge station (fig. 1) [25].

The collocation interpolation method, utilizing the leave-one-out approach, is then employed to estimate residuals at co-located points; for each point, it is temporarily excluded from the dataset, and the remaining points are used to interpolate its residual value. Application of the collocation method requires an empirical covariance function, which is derived by fitting the residuals of the co-located points with a Gauss–Markov model. The empirical and fitted covariance functions are presented in fig. 4.

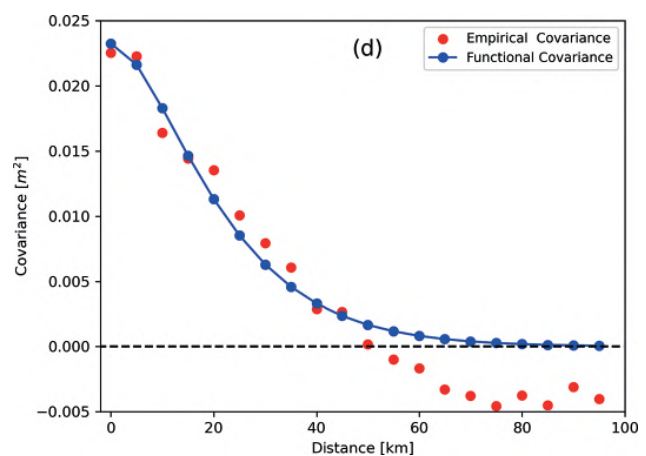


Fig. 4. Empirical and Gauss Markov fitting covariances computed from co-located points

Using the workflow presented in fig. 2, corrector surface-based height anomalies for co-located points are determined by combining the GGM-based height anomaly, the model-fitting height anomaly difference, and the interpolated residual at each co-located point. The difference between these estimated values and the GNSS/leveling height anomalies serves as an indicator of the model's accuracy. Fig. 5 shows the GNSS/leveling height anomalies (a), the corrector surface-based height anomalies (b), and their differences (c).

The statistics of the GNSS/leveling height anomalies, the corrector surface based height anomalies, and their differences are summarized in table 4, including the mean, standard deviation, maximum, and minimum values. The distribution of the height anomaly differences is illustrated in fig. 6. The results show that applying

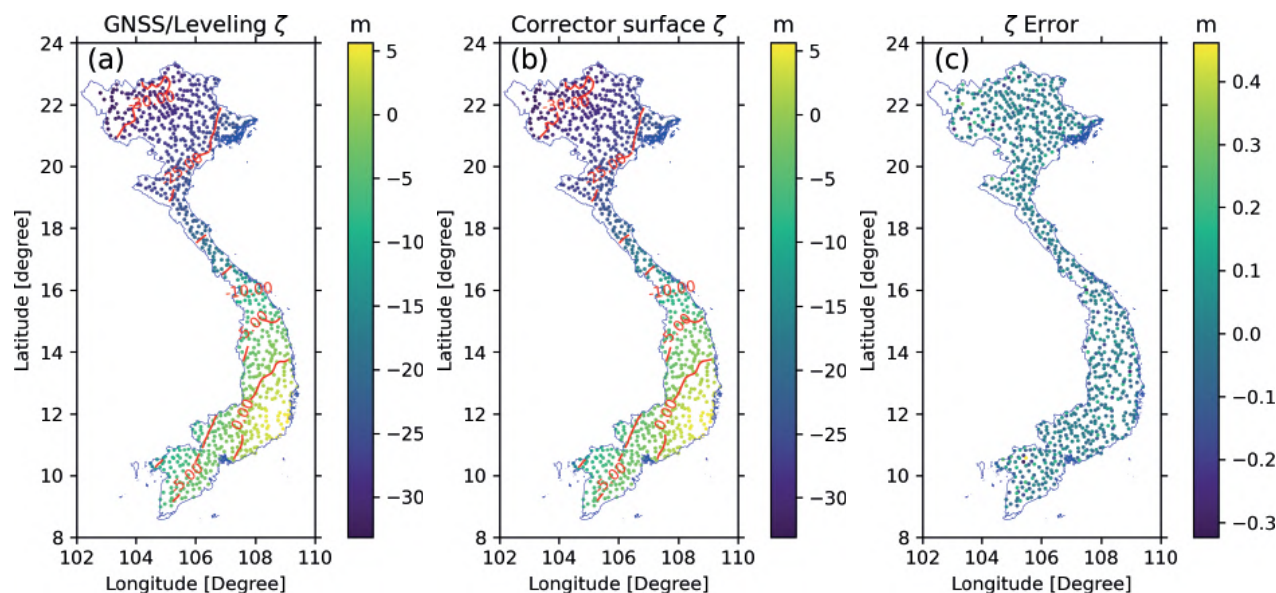


Fig. 5. Absolute height anomaly errors over the co-located GNSS/leveling points

the corrector surface reduces the standard deviation of the height anomaly differences to ± 0.083 m, compared with ± 0.153 m obtained without the corrector surface (table 3, 2nd row).

Table 4. Statistics of GNSS/leveling height anomalies, corrector surface based height anomalies, and their differences. Unit: m

Component	Mean	Std Dev	Max	Min
GNSS/leveling height anomaly $\zeta_{GNSS/lev}$	-14.611	± 12.275	5.601	-33.168
Corrector surface based height anomaly ζ_{cor}	-14.686	± 12.298	5.609	-33.102
Height anomaly difference	0.001	± 0.083	0.461	-0.323

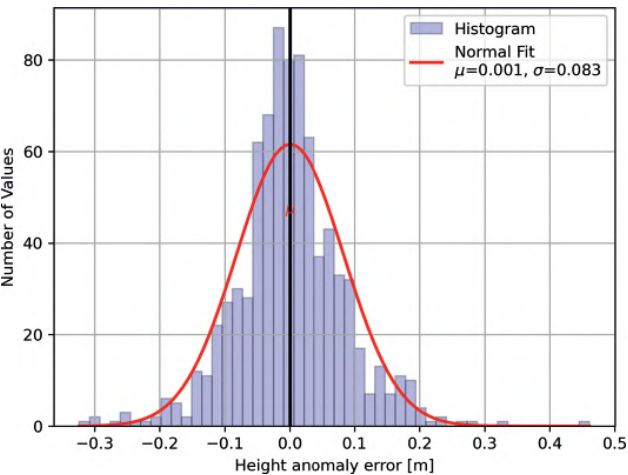


Fig. 6. Histogram of height anomaly errors

Corrector Surface Generation at Regular Grids

In this section, we create a corrector surface at regular grids of 5 arcminutes relying on the entire 816 co-located GNSS/leveling points (fig. 1), following the workflow in fig. 2. First, GGM-based height anomalies are estimated for all grid points relying on the best fit model XGM2019e5540 up to d/o 5540 with the results shown in fig. 7, a and statistical summary, including mean, standard deviation, maximum, and minimum values, shown in table 5 (2nd row). Then, the four-parameter deterministic Helmert transformation model derived from Section 4.2 at co-located points is applied to estimate model differences at 5-arcminute grids shown in fig. 7, b and table 5 (3rd row).

The statistical summaries of $\Delta\zeta_{model}$ shown in table 5 (3rd row) for 5-arcminute grid points are identical with those of co-located points (table 3, 3rd row) indicating that the co-located GNSS/leveling points are well-distributed throughout the Vietnam’s mainland. Subsequently, the collocation model developed in Section 4.2 is applied to interpolate residuals at the grids with the results shown in fig. 7, c and table 5 (4th row). Those three estimated components are finally sum together to get the corrector surface based height anomalies ζ_{cor} at the regular grids with (fig. 7, d and the 5th row of table5). These gridded values are then ready to be used for other applications in which ellipsoidal height can be transformed into leveling height according to the Vietnam national height system.

Conclusions

This study has presented an easy-to-use and practical approach for GNSS-based leveling height

Table 5. Statistical summary of components used to generate a corrector surface at regular grids of 5 arcminutes. The GGM-based height anomalies are computed relying on the best fit model XGM2019e5540. Unit: m

Row	Component	Mean	Std Dev	Max	Min
2 nd	GGM-based height anomaly ζ^{GGM}	-15.381	± 12.661	5.518	-34.387
3 rd	$\Delta\zeta^{model}$	0.431	± 0.044	0.476	0.304
4 th	Collocation interpolated residual r^{int}	-0.004	± 0.135	0.466	-0.546
5 th	Corrector surface based height anomaly ζ^{cor}	-14.954	± 12.627	6.020	-33.946

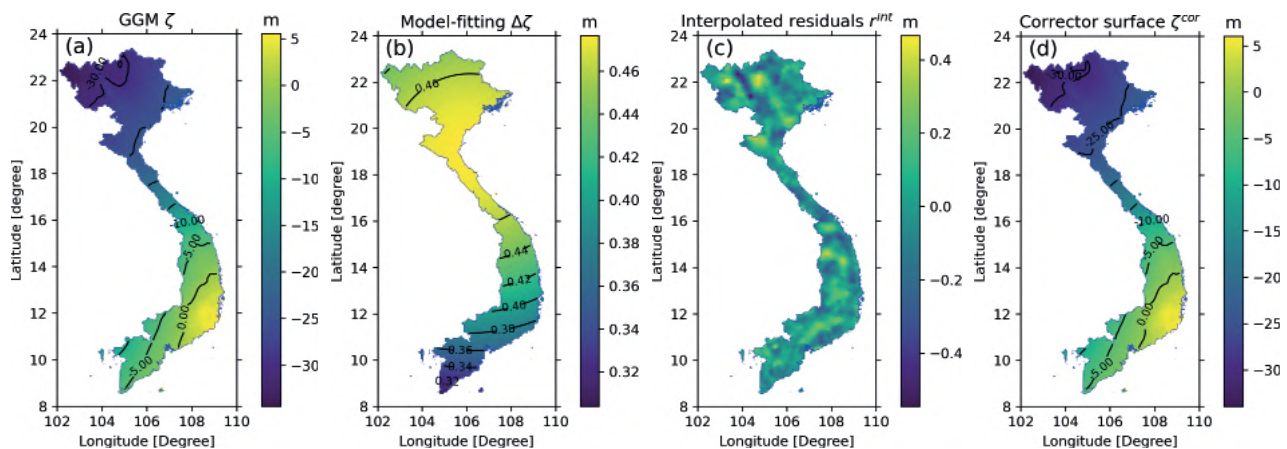


Fig. 7. Corrector surface generation at 5-arcminute grids: (a) XGM2019e5540-based height anomalies (ζ^{GGM}), (b) modeled height anomaly differences based on the four-parameter deterministic Helmert transformation model ($\Delta\zeta^{model}$), (c) residuals at gridded points interpolated from co-located points by the Collocation method, (d) corrector surface at gridded points computed as the combination of GGM ζ in (a), model-fitting $\Delta\zeta$ in (b) and interpolated residuals r^{int} in (c)

determination in Vietnam by integrating GGMs with existing GNSS/leveling data through a corrector surface methodology. Unlike conventional approaches that rely on dense local gravimetric observations or the development of a dedicated national quasigeoid model [29–31], the proposed framework combines a high-degree global geopotential model, a four-parameter deterministic Helmert transformation, and least-squares collocation interpolation of residuals. This strategy significantly reduces complexity while maintaining a level of accuracy suitable for engineering and geodetic applications.

A comprehensive evaluation of several state-of-the-art global geopotential models demonstrates that the XGM2019e model provides the best overall performance over the Vietnamese mainland, particularly at high spherical harmonic degrees. When combined with the four-parameter deterministic fitting, systematic biases and long-wavelength discrepancies between the global model and the national GNSS/leveling height system are effectively mitigated. The subsequent application of least-squares collocation further accounts for remaining medium- and short-wavelength residuals, capturing local variations not represented in the

global model alone. Validation using 816 co-located GNSS/leveling points shows that the proposed corrector surface reduces the standard deviation of height anomaly differences from ± 0.153 to ± 0.083 m, confirming a substantial improvement in accuracy.

The resulting corrector surface, generated at a regular grid resolution of 5 arcminutes, provides a consistent and efficient means to transform GNSS-derived ellipsoidal heights into leveling heights compatible with the Vietnam national height system. This product is particularly valuable for practical applications where rapid and reliable height determination is required. Moreover, the proposed methodology is flexible and future-proof in that it can be readily updated as new global geopotential models become available or as additional GNSS/leveling data are incorporated, without the need for extensive reprocessing.

Overall, the findings demonstrate that a carefully selected global geopotential model, when augmented by a simple deterministic transformation and statistical interpolation of residuals, can serve as a robust alternative to local gravimetric quasigeoid models in regions with limited gravity data coverage. The approach developed in this study offers a cost-effective and scalable solution for GNSS-based height determination.

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Sustainable development
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