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A new pseudo-static loading scheme for the hyperstatic reaction method - case of sub-rectangular tunnels under seismic conditions

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ABSTRACT

In recent years, many different methods for calculating the structural forces caused by earthquakes to be created in circular and rectangular tunnels have been devised. However, it is still unclear how sub-rectangular tunnels would behave when subjected to seismic loadings. A new pseudo-static loading scheme for sub-rectangular tunnels using the Hyperstatic Reaction Method (HRM) is the main topic of the current paper. In order to depict the interaction between the soil and the tunnel lining, new equations that allow for the computation of applied active loading as well as a variable spring stiffness coefficient are developed. Through a numerical study that takes into account a variety of seismic magnitudes, soil parameters, lining thickness, tunnel dimension, tunnel geometries, and tunnel depth, the proposed loading scheme is calibrated and validated. The comparisons demonstrate the effectiveness of the developed HRM method for the preliminary seismic design of sub-rectangular tunnels.

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1. Introduction

To meet the demands of densely populated urban regions for infrastructure expansion, tunnel construction has increased. When compared to surface constructions, tunnels are less vulnerable than surface-ground constructions. Nevertheless, earthquakes caused significant damage to tunnels, and it became clear that more trustworthy seismic-proof tunnel designs were needed (Hashash, Hook, Schmidt, et al., 2001; Zhang, Jiang, & Maegawa, 2020). Several analytical solutions (Kontoe, Avgerinos, & Potts, 2014; Lu, Chen, Chang, et al., 2018; Park, Tantayopin, Tontavanich, et al., 2009; Penzien & Wu, 1998; Penzien, 2020; Wang, 1993), numerical analysis (Amorosi & Boldini, 2009; Gu, Wang, Xu, et al., 2011; Kamal, Hussein, & Akl, 2021; Kiryu, Murono, & Morikawa, 2012; Patil, Choudhury, Ranjith, et al., 2018; Silva, Fabozzi, Nikitas, et al., 2021; Sun & Dias, 2019; Sun, Dias, & Sousa, 2020; Tsinidis, Pitilakis, & Madabhushi, 2016; Tsinidis, Silva, Anastasopoulos, et al., 2020), and experimental research (Cheng, Zhou, Liu, et al., 2021; Cilingir & Madabhushi, 2011; Ge, Wu, Ding, et al., 2021; Lanzano, Bilotta, Russo, et al., 2015) can be used to accomplish tunnel seismic designs. The analysis of the literature reveals that the circular and rectangular shapes of the tunnel are the main focus of studies analyzing seismic loading behavior in tunnels.

Additionally, the features of the ground, the tunnel lining, the interaction between the lining and the ground, and most crucially, the geometry of the tunnel, have a significant impact on how the tunnel behaves under seismic circumstances (Afshani, Akagi, & Konishi, 2020; Hashash, Hook, Schmidt, et al., 2001; Penzien, 2020; Tsinidis, Pitilakis, & Madabhushi, 2016; Tsinidis, Silva, Anastasopoulos, et al., 2020; Wang, 1993).

When underground constructions are excavated at shallow depths in urban areas, circular and rectangular tunnels are typically utilized. A special shape, namely sub-rectangular that combines rectangular and circular shapes, has recently been shown to be a great alternative. In comparison to circular tunnels, this shape allows for improved cross-section utilization efficiency. It also helps to prevent stress concentration at the corners of the rectangular tunnel. However, the newly shaped tunnels have not yet been thoroughly studied, particularly when exposed to seismic loads (Afshani, Akagi, & Konishi, 2020; Naggar, Hinchberge, & Hesham, 2008).

The Hyperstatic Reaction Method (HRM) is a finite element method and was used to compute the structural forces developed in the tunnel lining (Do, Dias, Oreste, et al., 2015; Do, Dias, Zhang, et al., 2020; Du, Dias, & Do, 2020; Kroetz, Do, Dias, et al., 2018; Oreste, 2007).

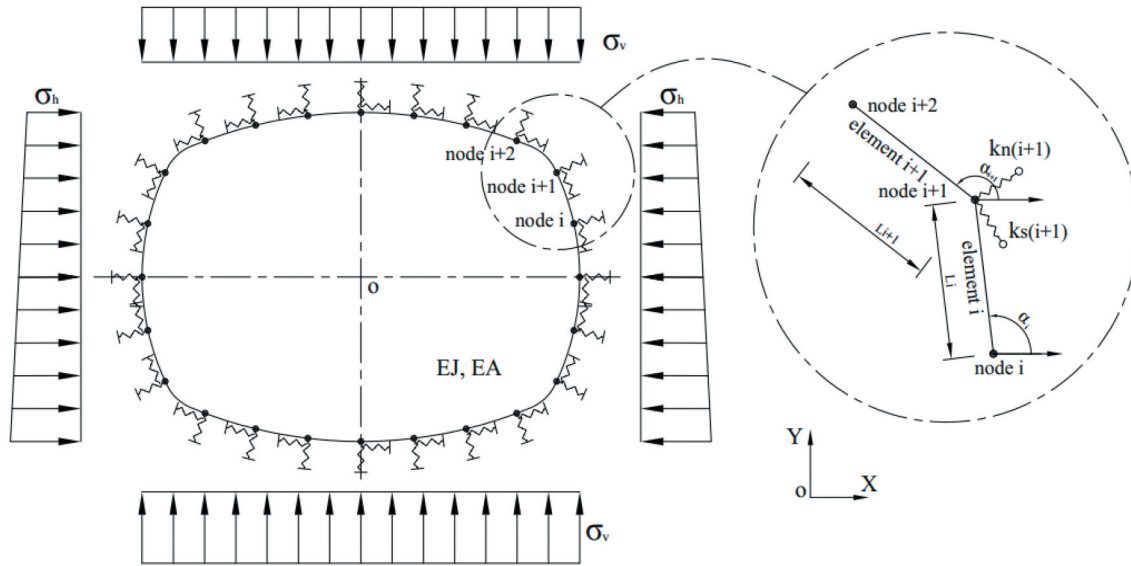


Figure 1. Calculation scheme of support structures using HRM method under static conditions. Where σ_v : the vertical loads; σ_h : the lateral loads; k_n : springs' normal stiffness; k_s : springs' shear stiffness; EJ and EA : bending and normal stiffness of the structure; X and Y are the global Cartesian coordinates (Do, Dias, Zhang, et al., 2020).

The HRM method is acknowledged as being ideal for optimization design procedures because it enables calculations to be completed in a fairly timely manner (Kroetz, Do, Dias, et al., 2018; Sun, Dias, & Sousa, 2020; Sun, Du, & Dias, 2021).

The HRM technique was successfully used to evaluate the seismic-induced structural forces in a circular tunnel lining taking into account pseudo-static conditions (Do, Dias, Oreste, et al., 2015; Sun, Du, & Dias, 2021). (Do, Dias, Oreste, et al. (2015)) used a set of dimensionless parameters to alter the external loading magnitude of the seismic-induced shear stresses based on the method proposed by (Penzien and Wu (1998), Naggar, Hinchberge, and Hesham (2008)) and (Sun, Du, and Dias (2021)) used a dimensionless factor to accurately represent the ground-tunnel interaction to take into account the effect of the ground-tunnel interaction on the applied external loadings.

In order to estimate seismic-induced structural forces in sub-rectangular tunnels in homogeneous isotropic grounds using the HRM method, a new pseudo-static loading scheme that acts on the tunnel lining is being introduced in the current study. First, this method's mathematical formulation is introduced. The numerical HRM process is then demonstrated in seismic conditions. A new external loading scheme applied to the sub-rectangular tunnels while taking into account a pseudo-static condition is proposed. Three dimensionless parameters are introduced to include the applied active loads and the

ground-tunnel interactions. Once the HRM solution has been calibrated using numerical calculations, the constitutive equations utilizing these parameters are given. Finally, a thorough validation of the established HRM approach is carried out while taking into account a variety of case scenarios, such as various seismic magnitudes, ground characteristics, lining thickness, tunnel geometry, tunnel dimensions, and tunnel depth. The study's novelty and scientific contribution is to provide an alternative approach to the highly effective seismic design of sub-rectangular tunnels.

2. Hyperstatic reaction method

2.1. Overview of HRM method

The HRM method was used for the analysis of segmental and continuous tunnel linings under static loads (Du, Dias, & Do, 2020; Oreste, 2007). Recently (Do, Dias, Zhang, et al. (2020)), have developed HRM method to investigate the behaviour of square and sub-rectangular tunnels under static loading. The authors proposed a calculation scheme of supporting structures and applied HRM for static conditions as presented in Figure 1.

Details of the mathematical formulation of the HRM method applied in sub-rectangular tunnels are thus not discussed and described in detail in the present study. Readers less familiar with the topic are advised to refer

to research available in the literature (Do, Dias, Oreste, et al., 2015; Do, Dias, Zhang, et al., 2020; Du, Dias, & Do, 2020; Kroetz, Do, Dias, et al., 2018; Oreste, 2007).

2.2. HRM method under seismic loadings

It is essential to characterize the external loads acting on the tunnel lining when utilizing HRM for seismic tunnel design. For circular tunnels subject to seismic loadings, it is assumed that the ovaling deformation of the cross-section is the most important one (Hashash, Hook, Schmidt, et al., 2001; Sun & Dias, 2019), as seen in Figure 2a. The ovaling deformation can be simplified to a pseudo-static situation in two-dimensional plane strain circumstances (Do, Dias, Oreste, et al., 2015; Park, Tantayopin, Tontavanich, et al., 2009). Therefore, when

the external shear stresses are applied at the far-field boundary, as shown in Figure 2b, seismically generated stresses and deformation can be easily determined. The shear stress, τ , can be estimated using the free-field shear strain γ_c (Hashash, Hook, Schmidt, et al., 2001; Penzien & Wu, 1998):

$$\tau = \frac{E_s \gamma_c}{2(1 + \nu_s)} \quad (1)$$

where γ_c is the maximum shear strain; E_s is Young's modulus of the ground, and ν_s is Poisson's ratio of the ground.

Similarly, when applying shear stresses at the far-field boundary, the critical state of sub-rectangular tunnels subjected to seismic loading causes a sub-ovaling deformation of the tunnel lining as seen in Figure 2c. This

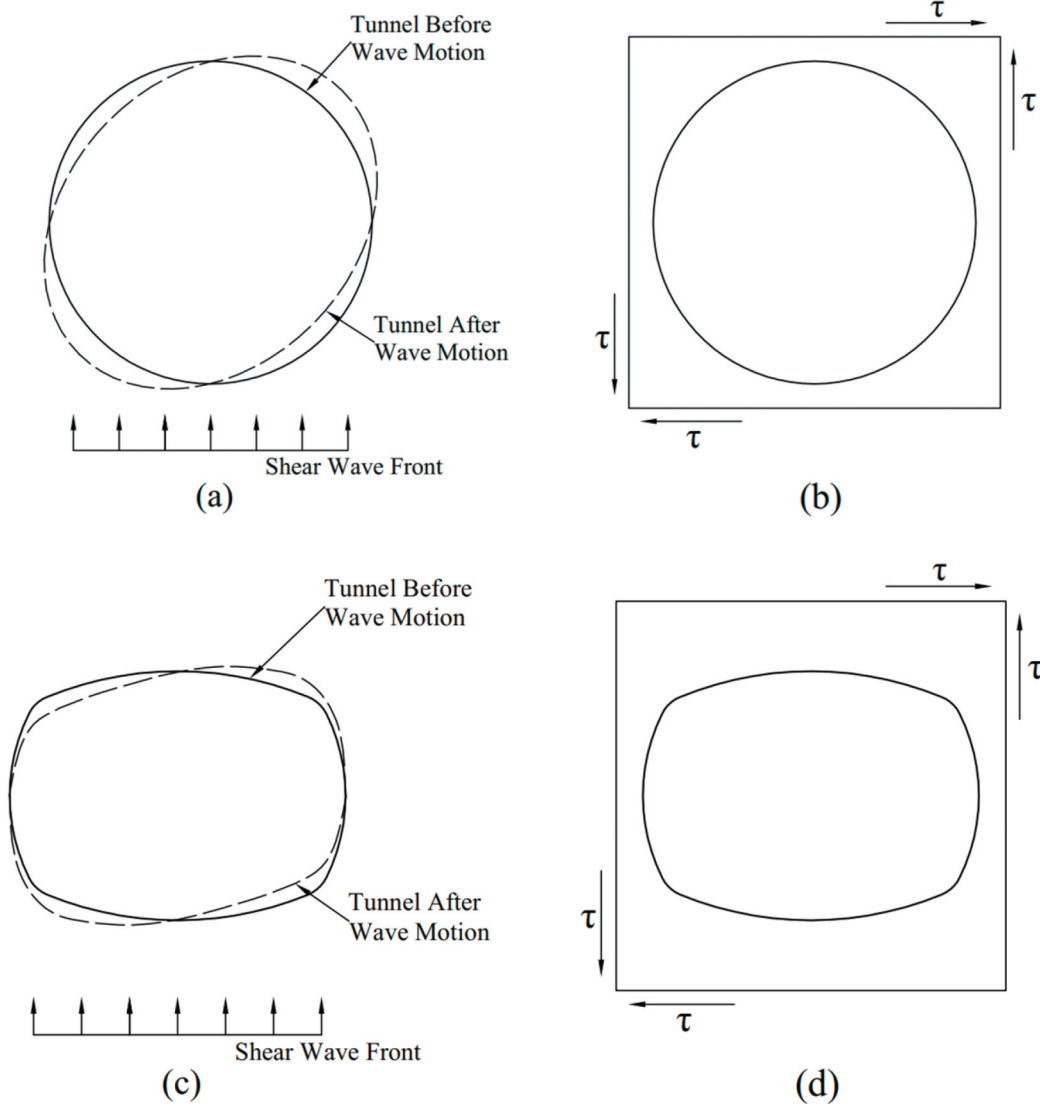


Figure 2. Transversal response in 2D plane strain conditions for circular tunnels (a) ovaling deformation; (b) seismic shear loading; (c) sub-ovaling deformation; (d) seismic shear loading.

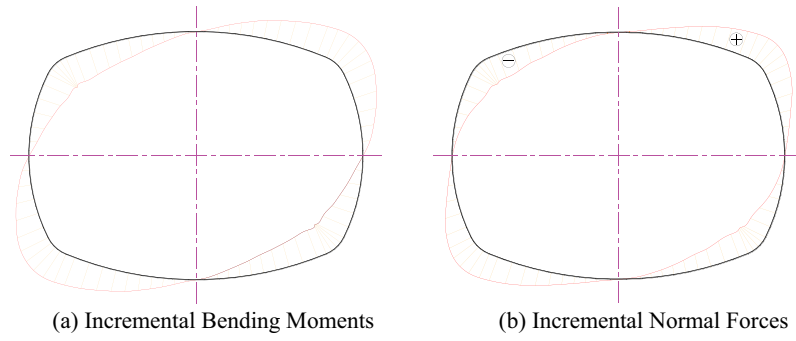


Figure 3. Incremental bending moments and normal forces of sub-rectangular tunnels obtained using FDM model. (a) Incremental bending moments, (b) Incremental normal forces.

result is obtained by using a finite-difference model (FDM) and incremental structural forces are presented in Figure 3.

Instead of the shear stresses applied at the far-field boundary, external loads acting directly along the tunnel lining are assumed in the HRM model to consider the seismic loading (Do, Dias, Oreste, et al., 2015). One of the main purposes of this study is to propose an external loading scheme applied to the tunnel lining as presented in Figure 4. This is to obtain a sub-ovaling deformation as observed at the critical state of sub-rectangular tunnels subjected to seismic loading. Point A is the intersection between the sidewall and shoulder, point B is located at the center of the shoulder parts, and point C is the intersection between the crown and shoulder as seen in Figure 4.

As formerly mentioned, induced structural forces and deformations of tunnel linings are mainly influenced by the external loads and ground-lining interactions considering the initial spring's stiffness. It is necessary to note that, in the HRM method, the normal springs can only be active in compression when the tunnel lining is moving toward the surrounding ground. The compressive external loads applied to the tunnel in a certain direction will automatically cause tensile loads in the perpendicular direction (Do, Dias, Oreste, et al., 2015; Sun, Du, & Dias, 2021). That is why a couple of parameters (a) and (b) are essential to be used to adjust the external loads applied to the sub-rectangular tunnel lining during a seismic event, as indicated in Figure 4.

In the HRM, the interaction between ground and tunnel lining is simulated through normal and tangential springs connected to the lining structure's nodes

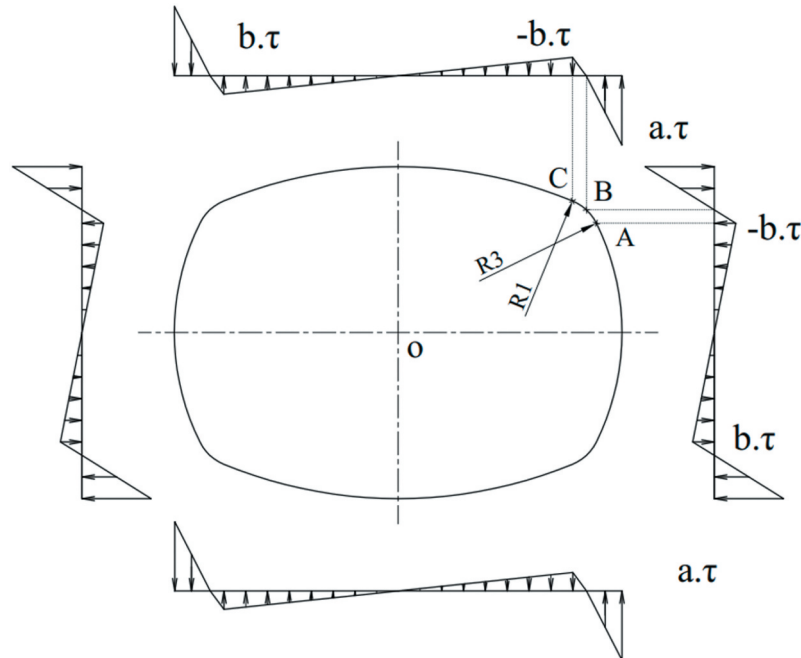


Figure 4. Equivalent pseudo-static external loading in the HRM method for sub-rectangular tunnels.

(Figure 1) which are, respectively, characterized by k_n and k_s and estimated by the ground initial stiffness η_0 . In sub-rectangular tunnels, the radius of lining parts varies along the tunnel periphery, the initial stiffness of the ground η_0 will then change depending on the radius (Do, Dias, Oreste, et al., 2015):

$$\eta_{n,0} = \beta \frac{1}{1 + \nu_s} \cdot \frac{E_s}{Ri} \quad (2)$$

Where E_s and ν_s are, respectively, the ground Young's modulus and Poisson's ratio; Ri is the radius of part i ($i = 1, 2$ and 3 corresponding to the crown, shoulder and sidewall of the tunnel boundary); β is a dimensionless factor.

In static analyses, the value of the dimensionless factor (β) which affects the spring stiffness was usually set to 1 or 2 (Do, Dias, Oreste, et al., 2015). Recently (Sun, Dias, and Sousa (2020), and (Sun, Du, and Dias (2021)) estimated the β value, depending on the properties of the ground and tunnel lining for the case of circular tunnels subjected to seismic loading. In the present work, a variation of the dimensionless factor (β) is also used to veritably represent the ground-tunnel lining interaction.

A numerical parametric analysis is conducted in this study to estimate the value of the three dimensionless parameters (i.e., a , b and β). After that, the results attained from the HRM method are verified against the ones given by the FDM model, considering a large range of ground properties, lining characteristics and tunnel geometry.

3. Numerical implementation

The model implementation has been carried out using Flac3D (Itasca Consulting Group Itasca, 2012), a finite difference code which was successfully applied in previous studies (Do, Dias, Oreste, et al., 2015; Pham, Do, & Dias, 2021). In this section, a brief description of the numerical model developed is first described. The model was adopted to calibrate the three dimensionless

parameters used in the HRM method. Then, the numerical procedure to execute the HRM in the case of sub-rectangular tunnels subjected to seismic loadings is presented.

3.1. FDM numerical simulations

Using Flac^{3D}, a 2D plane strain model is created by setting a single layer of zones in the y -direction along the tunnel's longitudinal axis (Figure 5). The ground-mass is discretized into hexahedral zones and the dimension of the zones increases when moving far away from the tunnel boundary. The numerical model is 120 m wide in the x -direction, and 40 m high in the z -direction and composed of approximately 5816 zones and 11,870 nodes. The dimensions of the numerical model were evaluated based on a parametric analysis to eliminate the potential effect of the width of the numerical model on the computed response of the tunnel when the seismic loading is simulated by means of a prescribed displacement pattern employed on the side and top boundaries of the numerical model. The model bottom is fixed in all directions, and the vertical sides are horizontally restrained.

The tunnel lining is modelled using embedded liner elements attached to the zone faces along the tunnel boundary with interfaces. It is well known that ground-tunnel interface conditions may significantly affect the lining response under seismic loading. In this study, the no-slip interface assumption is adopted, referring to the upper boundary condition, i.e., the worst case of stability (Tsinidis, Pitilakis, & Madabhushi, 2016). The normal stiffness k_n and tangential stiffness k_s of the lining-zone interface are determined based on a given rule-of-thumb (Itasca Consulting Group Itasca, 2012). k_n and k_s are set equal to 100 times the equivalent stiffness of the stiffest neighbouring zone for the no-slip condition case.

The geometric parameters of sub-rectangular tunnels are presented in Figure 6. Other ground and lining

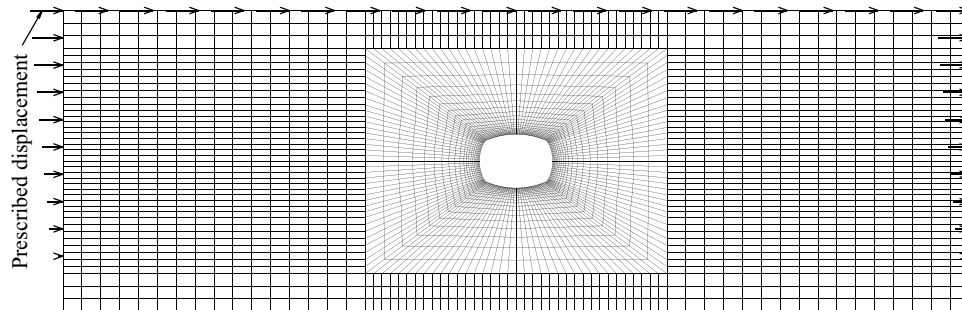


Figure 5. Geometry and pseudo-static loading conditions in the FDM model of a sub-rectangular tunnel.

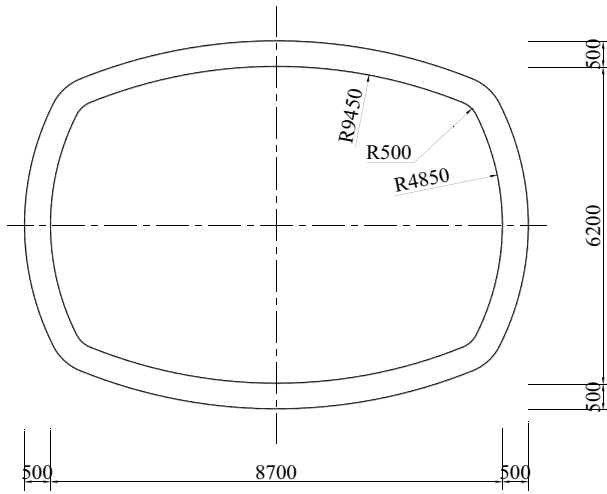


Figure 6. Cross section of sub-rectangular express tunnel in Shanghai (unit: mm) (Do, Dias, Zhang, et al., 2020).

parameters listed in Table 1 are adopted. It should be mentioned that for the calibration purpose of parameters a , b and β , Young's modulus of ground E_s is assumed to vary from 10 to 350 MPa, while the lining thickness varies between 0.3 and 0.8 m.

3.2. Numerical procedure in HRM method

To implement the HRM method in the case of sub-rectangular tunnels subjected to seismic loading, it is necessary to determine the formulas of the three dimensionless parameters (a , b and β) which define the external loadings applied on the tunnel lining. Extensive

comparisons between the seismic-induced incremental internal force predicted by the HRM method and the ones calculated using the FDM numerical model are conducted.

In the initial calibration phase, the parameters of Table 1 were adopted. Based on that, the parameters of tunnel dimensions are changed in a certain range to cover most of the cases that occurred. The tunnel width (w) is assumed to be modified from 9.7 m to 15.52 m to make wider tunnels that have a uniform shape with the reference tunnel illustrated in Figure 6. The t/w ratio, therefore, varied from 0.032 to 0.052, respectively. In addition, different sub-rectangular shapes, i.e., SR1 to SR4, with all dimension characteristics presented in Table 2 and Figure 7 are adopted. The maximum horizontal acceleration, $a_H = 0.5g$, corresponds to the shear strains, γ_c of 0.38%. The calibration process of the three parameters (i.e., a , b and β) can then be performed. The main procedure to calibrate the three parameters is illustrated in Table 3 and Figure 8.

Equations that reflect the impact of the three parameters on the ground, lining characteristics and tunnel dimensions can be developed once the calibration process is complete. Using the best fit, the formulas are suggested. The parameters β , a , and b can be given as follows:

$$\beta = \beta_1 + \beta_2 + \beta_3 + \beta_4 \quad (3)$$

$$\beta_1 = -1.65E_s^{-0.3} + 1.477 \quad (4)$$

Table 1. Input parameters of the reference case study.

Parameter	Symbol	Unit	Value or Range
Ground properties			
Unit weight	γ	kN/m ³	18
Young's modulus	E_s	MPa	10–350
Poisson's ratio	ν_s	-	0.34
Internal friction angle	ϕ	degrees	33
Cohesion	C_u	kPa	0
Lateral earth pressure coefficient	K_0	-	0.5
Depth of tunnel	H	m	20
Peak horizontal acceleration at the ground surface	a_H	g	0.5
Tunnel lining properties			
Young's modulus	E_l	GPa	35
Poisson's ratio	ν_l	-	0.15
Lining thickness	t	m	0.3–0.8
Tunnel height	h	m	7.2
Tunnel width	w	m	9.7

Table 2. Geometrical parameters of tunnel shape cases (Do, Dias, Zhang, et al., 2020).

Case	Tunnel width (w) (m)	Tunnel height (h) (m)	h/w ratio	R1 (m)	R2 (m)	R3 (m)
SR1	8.76	8.15	0.930	8.36	1.02	4.99
SR2	9.13	7.89	0.864	7.09	1.23	4.81
SR3	9.39	7.53	0.802	8.5	0.96	5.07
SR4 (reference case)	9.70	7.20	0.742	9.95	1.00	5.35

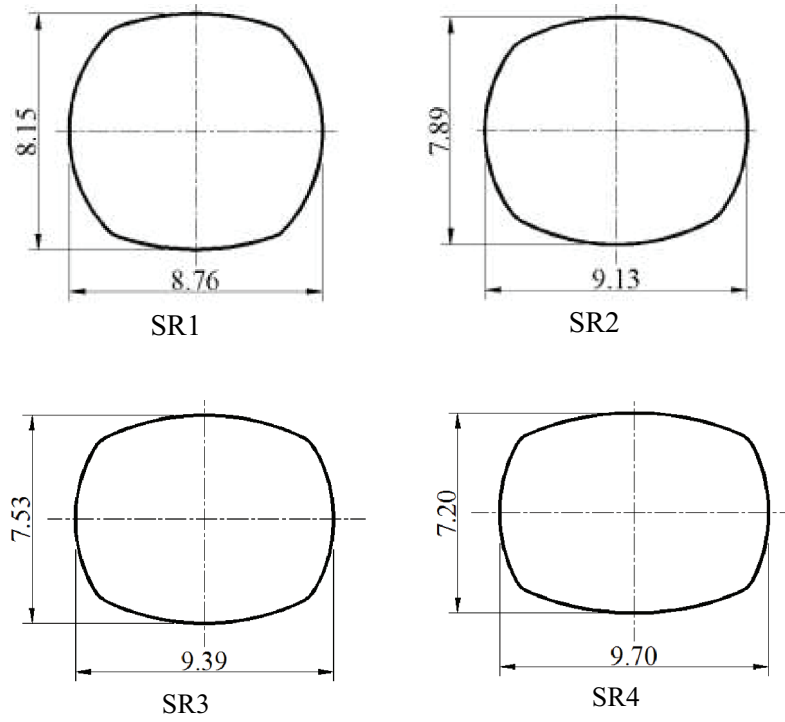


Figure 7. Shapes of tunnel cases (unit: m) (Do, Dias, Zhang, et al., 2020).

Table 3. Calibration process.

Step	Description
1	Using the specified parameter ranges provided in Tables 1 and 2, generate the input parameters $\{t_i, h_i, w_i, E_{si}\}$ for the ground, lining, and tunnel dimensions.
2	Estimating seismic-induced incremental axial forces and bending moments calculation $\{N_{FDM}, M_{FDM}\}$ using FDM model, and computing the initial values of $\{N_{HRM}, M_{HRM}\}$ using the HRM method based on $a=b=\beta=1$.
3	Calculating the relative inaccuracy of incremental axial forces and bending moments acquired using two different techniques.
4	If $e_N \leq 0.02$ and $e_M \leq 0.02$, export a, b and β . Otherwise, update these three parameters (i.e., a, b, β) until the target precision is reached.
5	Repeat steps 2 through 4 until all case situations with established parameter range from Tables 1 and 2 are taken into account.
6	Regression analysis is used to determine the formulas that describe a, b , and β as functions of t_i, h_i, w_i, E_{si} parameters.

$$\beta_2 = 4.8002 \left(\frac{t}{h} \right) - 0.3333 \quad (5)$$

$$b_2 = 23.04 \left(\frac{t}{h} \right) - 1.6 \quad (11)$$

$$\beta_3 = 30644 \left(\frac{t}{w} \right)^3 - 4452.3 \left(\frac{t}{w} \right)^2 + 207.88 \left(\frac{t}{w} \right) - 3.0828 \quad (6)$$

$$b_3 = -20461 \left(\frac{t}{w} \right)^3 + 3736 \left(\frac{t}{w} \right)^2 - 192.79 \left(\frac{t}{w} \right) + 2.8135 \quad (12)$$

$$\beta_4 = 7.9746 \left(\frac{h}{w} \right) - 5.9192 \quad (7)$$

$$b_4 = 5.3148 \left(\frac{h}{w} \right) - 3.945 \quad (13)$$

$$a = 12.155 + 6.45E_s^{-1} \quad (8)$$

$$b = b_1 + b_2 + b_3 + b_4 \quad (9)$$

$$b_1 = 10E_s^{-0.8} - 0.305 \quad (10)$$

While coefficient a is expressed as a function of the ground's Young's modulus alone (E_s), coefficients β and b are the functions of the lining thickness (t), tunnel height (h), tunnel width (w), and ground's Young's modulus (E_s), as presented in Figs. 9 and 10. In Figure 9b, the value of the parameter β_1 increases rapidly when E_s value increases from 10 to 150 MPa, and its value rises slightly when values of E_s are larger

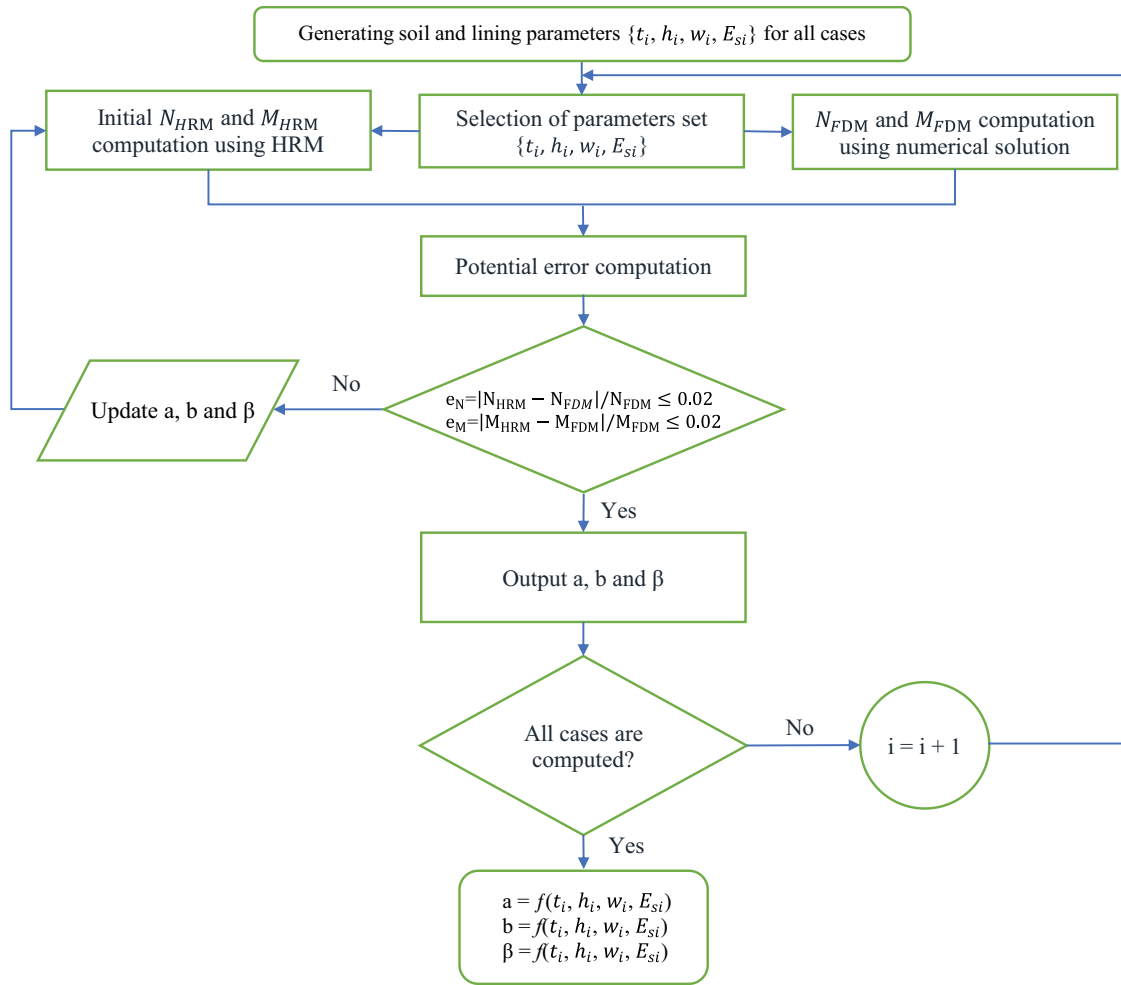


Figure 8. Calibration flowchart of the three parameters in HRM method.

than 150 MPa. However, for parameters a and b_1 in Figures 9a and 10a, respectively, there was a rapid decrease of these parameters when E_s values changed within a range of 10 and 100 MPa. When E_s values continue to increase, the values of these two parameters stay constant. Parameters $\beta_2, \beta_3, \beta_4, b_2, b_3$, and b_4 are greatly affected by t/h , t/w , and h/w ratios (Figure 9c–9e and Fig. 10b–10d).

It is interesting to note that for E_s value of 75 MPa and t/h ratio of 0.07 (Figures. 10a,b), b_1 and b_2 values are equal to zero and therefore do not cause additional seismic loading acting on the tunnel lining through the b value in Equation 9. In other words, when the tunnel structure is more flexible than the ground, the tunnel lining will amplify the distortion compared with the ground shear distortions in the free field. This case corresponds to E_s values larger than 75 MPa in Figure 10a, and the t/h ratios are smaller than 0.07 in Figure 10b and 10c. In contrast, when a tunnel lining is stiffer than the ground, corresponding to E_s values

smaller than 75 MPa in Figure 10a, the t/h ratio is larger than 0.07 in Figure 10b and 10d, the lining tends to resist the ground displacements.

To have a clear understanding of the results obtained by HRM and the numerical FDM model, Figure 11 introduces a comparative example of the incremental bending moments and normal force distribution in the sub-rectangular tunnel lining subjected to a seismic loading when $E_s = 100$ MPa and $t = 0.5$ m. Other ground parameters of the reference case presented in Table 1 are adopted. Figure 11 reveals insignificant differences between the extreme incremental structural forces obtained by the HRM method and the FDM model. The differences are 1.2% and 0.6% corresponding to the extreme incremental bending moments and the normal forces. The discrepancies between the incremental bending moments and normal forces observed at the top and bottom of the tunnel lining could be explained by the fact that the chosen equivalent external loading presented in Figure 4 is a simplified one.

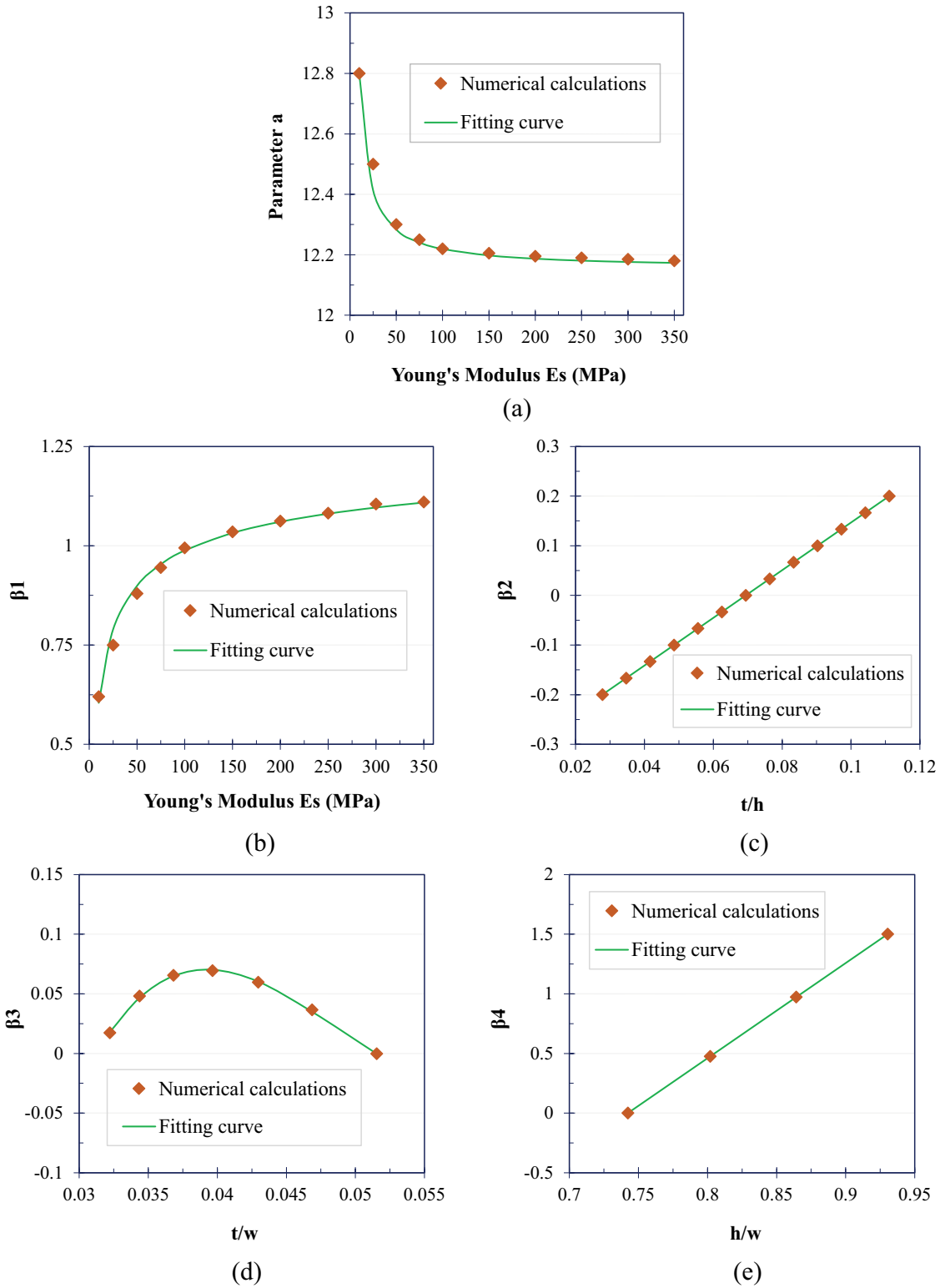


Figure 9. Obtained numerical results and fitting curves adopted for the parameters a and β_i .

4. Validation of the proposed HRM model

To show the applicability of the created HRM technique, thorough validations were conducted. The established HRM method's accuracy will be evaluated in the

first validation when a variety of peak horizontal seismic accelerations are taken into account (a_H). Then, validations 2 and 3 employ the variable Young's modulus of ground and lining thickness. While uniform tunnels

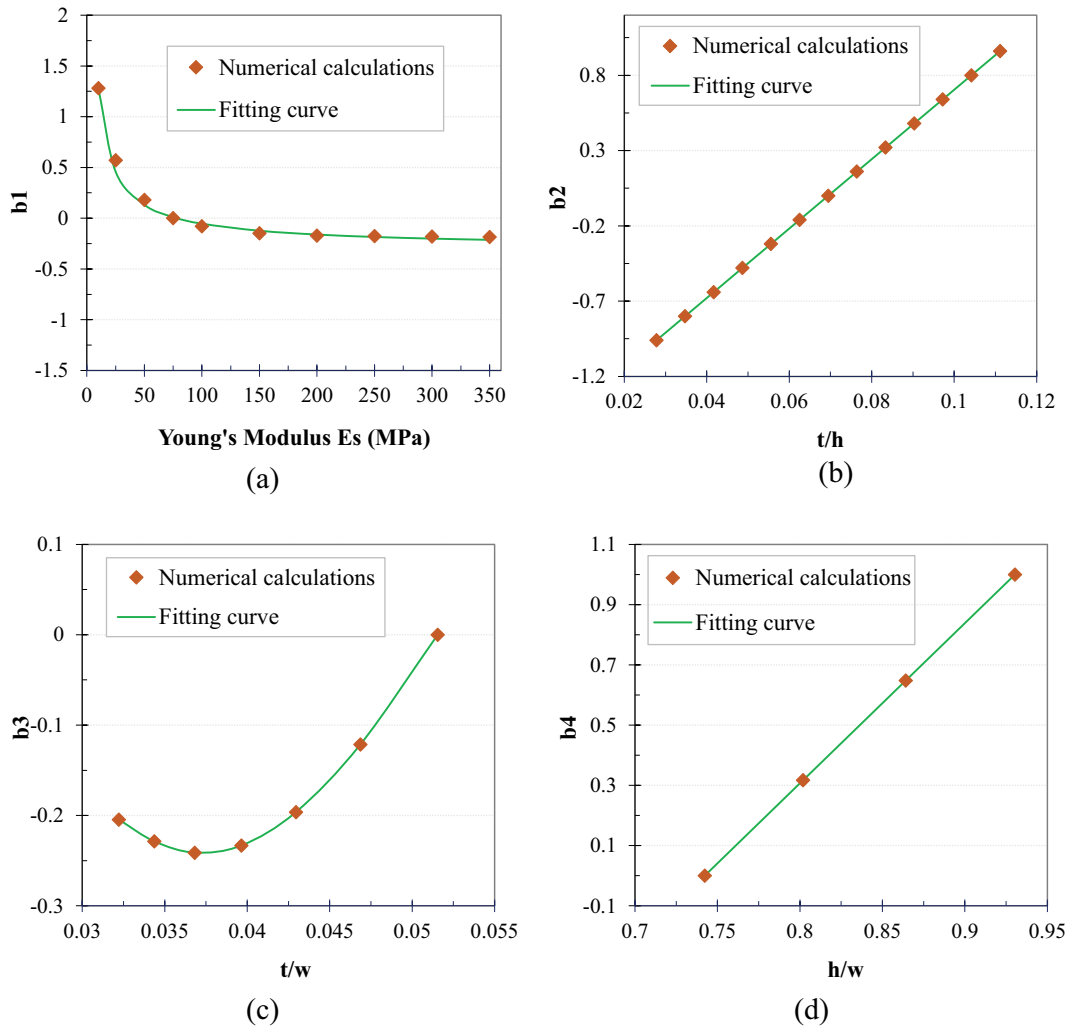


Figure 10. Obtained numerical results and fitting curves for parameters b_i .

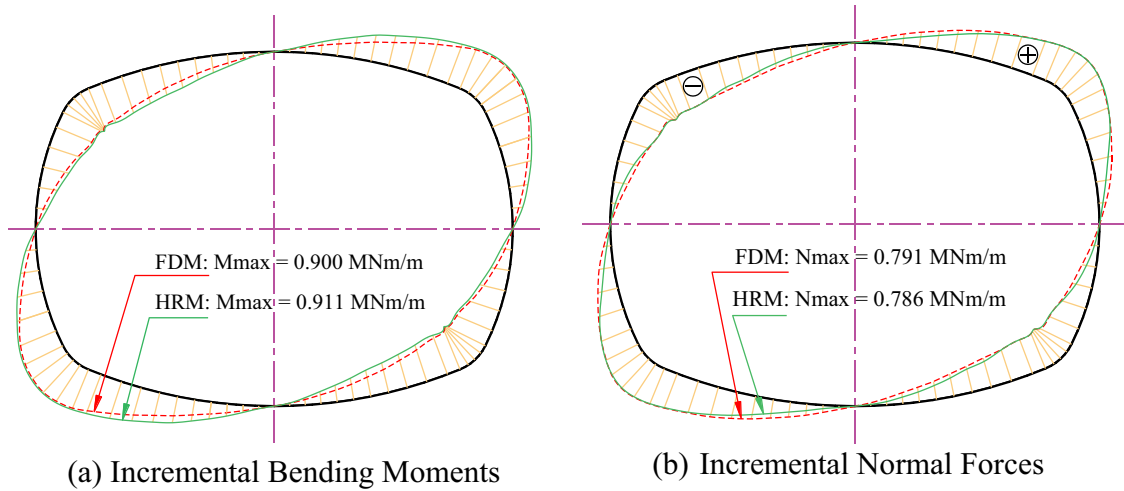


Figure 11. Comparison of the incremental bending moments and axial forces calculated by the developed HRM method and numerical FDM calculation.

with various cross-section areas are taken into account in validation 4, validation 5 makes use of several sub-rectangular shapes with geometrical parameters of tunnel shape examples from Table 2 (Do, Dias, Zhang, et al., 2020). In validation 6, it is taken into account how the tunnel burial depth affects the behavior of the tunnel lining. The final validation uses ground parameters from works by (Hashash, Park, and Yao (2005)) and (Sun, Du, and Dias (2021)) to accomplish the validation. In each validation, the seismic-induced incremental structural forces obtained by the HRM method are compared with the numerical FDM solution.

4.1. Validation 1

The formulas established for parameters a , b , and β and presented in section 3.2 are proposed based on the case of a_H of 0.5 g. In this particular validation, the maximum horizontal acceleration, a_H , is assumed to change in the range from 0.05 to 0.5 g, corresponding to the shear strains, γ_c , of 0.038% and 0.38%. The other parameters of the reference case presented in Table 1 are adopted. A comparison between the developed HRM method and the FDM calculations in terms of extreme incremental bending moments and normal forces is performed.

As shown in Figure 12, both the extreme incremental normal forces and bending moments computed using the developed HRM method show a very good agreement with the numerical FDM results. These two methods can capture the same trend, i.e., the absolute extreme incremental bending moments and normal

forces increase with the applied maximum horizontal acceleration (a_H) rise. The discrepancy is under 2.2% and 2% corresponding to the extreme incremental bending moments and normal forces. It proves that the developed HRM method can be effectively applied in a large range of horizontal accelerations, a_H .

4.2. Validation 2

Validation 2 is conducted for the ground Young's modulus variation in a range from 10 to 350 MPa. The lining thickness equals 0.5 m and the other ground parameters, based on the reference case study and listed in Table 1, were used.

Figure 13 shows a very good agreement of the extreme incremental bending moments and normal forces developed in the tunnel lining calculated using HRM method and the numerical FDM when considering the E_s changes. The extreme incremental bending moments are strongly dependent on the E_s values as seen in Figure 13(a). With E_s values smaller than 50 MPa, a decrease of E_s induces a decrease in the absolute extreme incremental bending moments. When the E_s value is greater than 50 MPa, an increase of the E_s value causes a decrease of the absolute extreme incremental bending moments. Compared to the numerical FDM results, the HRM method precisely predicts the extreme incremental bending moments. For almost all the E_s values, the difference is smaller than 3.5%. Except for the case of an E_s value equal to 25 MPa, the discrepancies are equal to 6.8% and 7.2% for the maximum and minimum incremental bending moments, respectively. The observation is that it could be related to the accuracy of the fitting curves of the parameters a , b and β (Figures 9 and 10).

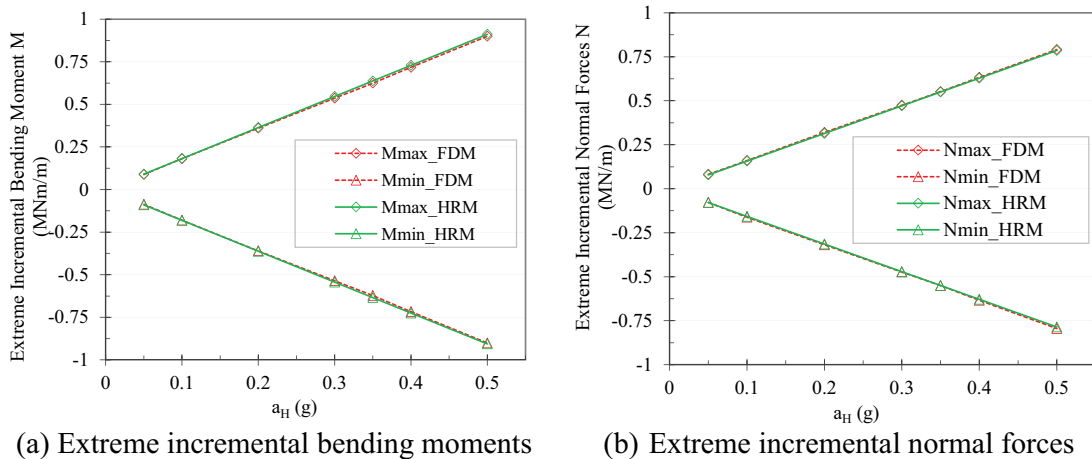


Figure 12. Horizontal accelerations a_H impact on extreme incremental structural forces of the sub-rectangular tunnel lining.

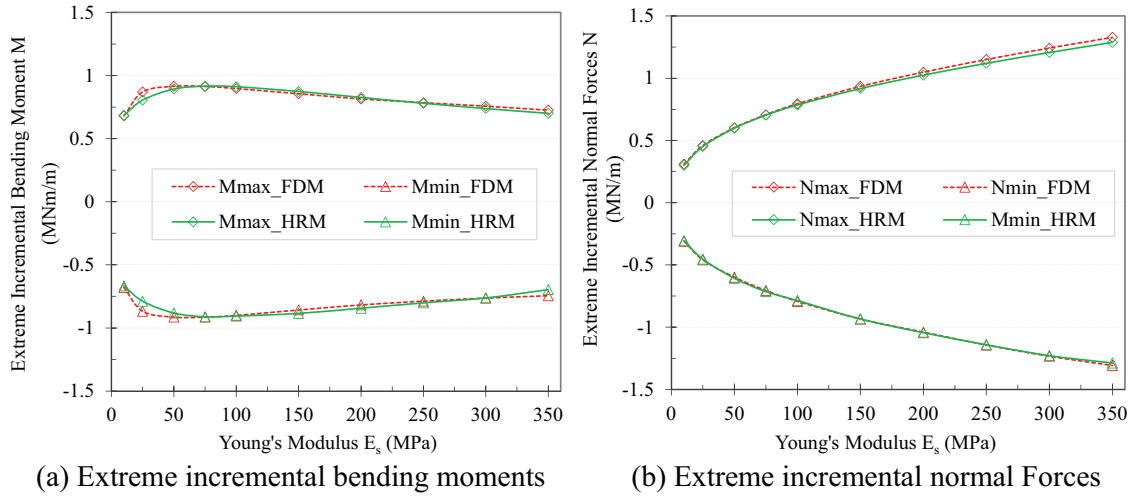


Figure 13. Effect of E_s changes on the extreme incremental structural forces of the sub-rectangular tunnel lining.

Figure 13(b) shows that an increase of E_s can cause a rapid increase of the absolute extreme incremental normal forces in the tunnel lining. The extreme incremental normal forces obtained by the developed HRM method are matching the numerical FDM results perfectly, the differences are less than 3% when E_s values change from 10 to 350 MPa.

4.3. Validation 3

The tunnel lining thickness is assumed to vary between 0.3 to 0.8 m while the other parameters are based on the reference case described in Table 1 and E_s is equal to 100 MPa. The comparisons between the HRM method and the numerical FDM ones are given in Figure 14, in terms

of extreme incremental bending moments and normal forces.

The extreme incremental structural forces calculated using the HRM method exhibit the same trend as those using numerical computations. Both the absolute extreme normal forces and bending moments increase linearly with the lining thickness increase. It should be noted that the incremental normal forces variations caused by the lining thickness increase are less significant when compared to the incremental bending moment ones (Figure 14(a) and 14(b)). Figure 14 indicates that the extreme incremental structural forces determined by the developed HRM method agree well with the numerical FDM computations. The discrepancy is under 2% for both the extreme incremental bending moments and normal forces.

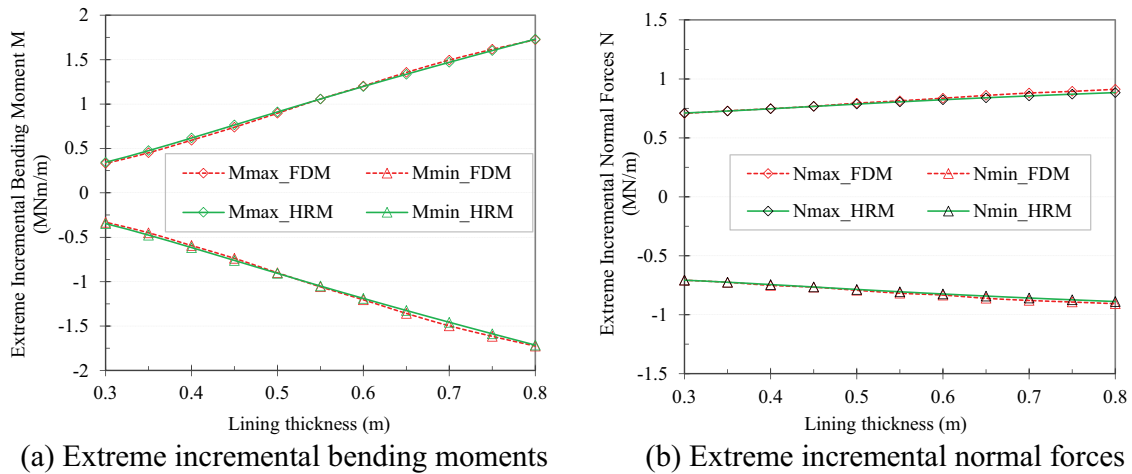


Figure 14. Effect of the lining thickness on the extreme incremental structural forces of the sub-rectangular tunnel lining.

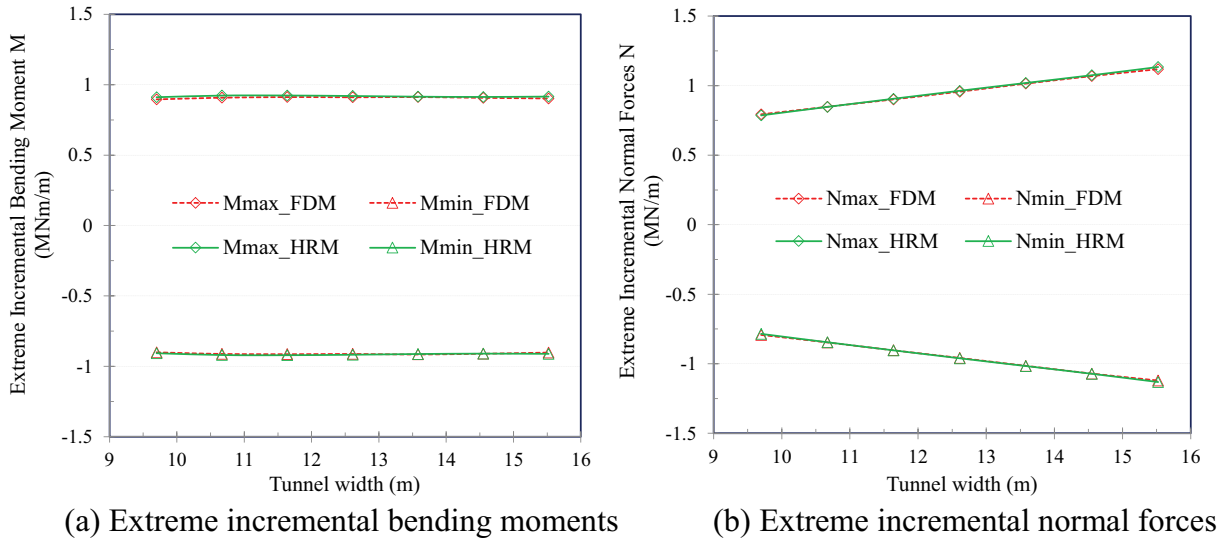


Figure 15. The cross-section dimensions influence on the extreme incremental structural forces of the sub-rectangular tunnel lining.

4.4. Validation 4

In this validation process, the sizes of the uniform tunnel cross-sections change from 1.0 to 1.6 times compared to the original one (Figure 6), corresponding to the tunnel cross-section width (w) changes from 9.7 to 15.52 m. The other parameters are adopted as shown in Table 1, although E_s equals 100 MPa. The extreme incremental structural forces obtained by HRM compared with the numerical FDM computations are illustrated in Figure 15.

In terms of the extreme incremental structural forces, the results presented in Figure 15 show a very good agreement between the HRM method and the numerical modelling using FDM. Figure 15(a) also indicated that the extreme incremental bending moments are almost constant when the tunnel width increases from 9.7 to 15.52 m. The difference between the extreme bending moments obtained using the developed HRM method and the numerical computations is lower than 1.7%. In contrast to the extreme incremental bending moments, the absolute extreme incremental normal forces tend to increase linearly as the tunnel width increases. The discrepancy between the extreme normal forces calculated by the HRM method and the numerical FDM ones is under 1.4% (Figure 15(b)). When the tunnel width changes, the HRM method shows very good efficiency too.

4.5. Validation 5

Another step was to validate the efficiency of the developed HRM method with the different tunnel geometry presented in Table 2 (Do, Dias, Zhang, et al., 2020), while

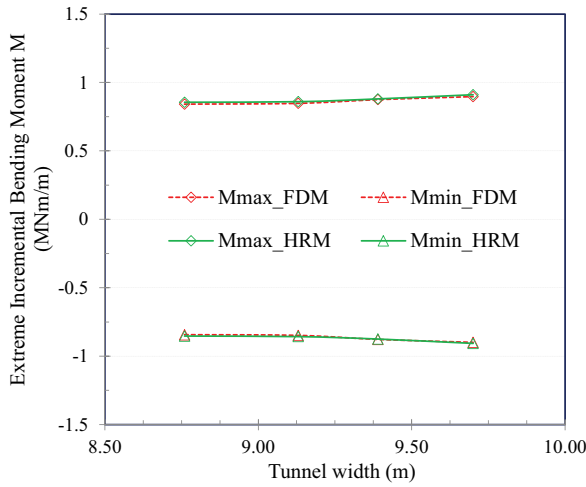
the other parameters are as shown in Table 1 ($E_s = 100$ MPa, $t = 0.5$ m). The results of the extreme incremental structural forces obtained by HRM compared with the numerical FDM ones are shown in Figure 16. It should be mentioned that the tunnel widths are used in Figure 16 to distinguish different tunnel geometries.

Figure 16 indicates how the tunnel shapes influence the extreme incremental normal forces and bending moments. The extreme incremental bending moments increase slightly by 6% when the tunnel shape changes from SR1 to SR4. In contrast, the extreme incremental normal forces are almost constant. The extreme incremental normal forces in the tunnel lining obtained by the HRM method are matching well the numerical computations using FDM, the discrepancy is lower than 3% for all the cases. Compared with the numerical results, the HRM method allows accurate prediction of the extreme incremental bending moments, and the differences in all cases are under 2%.

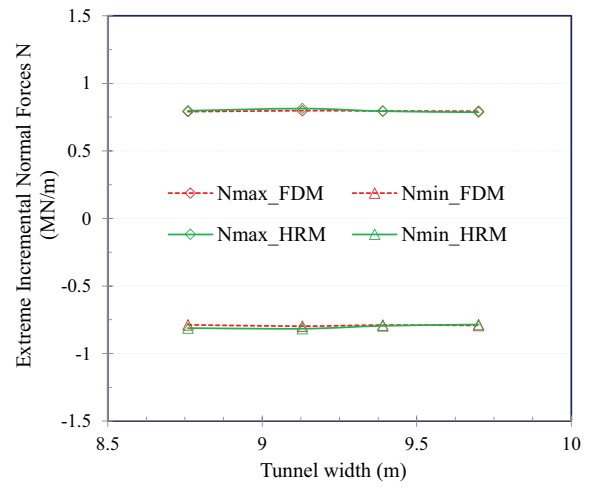
4.6. Validation 6

Validation 6 is conducted considering the tunnel burial depth variation in the range from 5 m to 20 m. The other parameters of the tunnel lining and ground are based on the reference case study listed in Table 1 ($E_s = 100$ MPa, $t = 0.5$ m). The results of the extreme incremental structural forces obtained by HRM compared with the numerical FDM ones are shown in Figure 17.

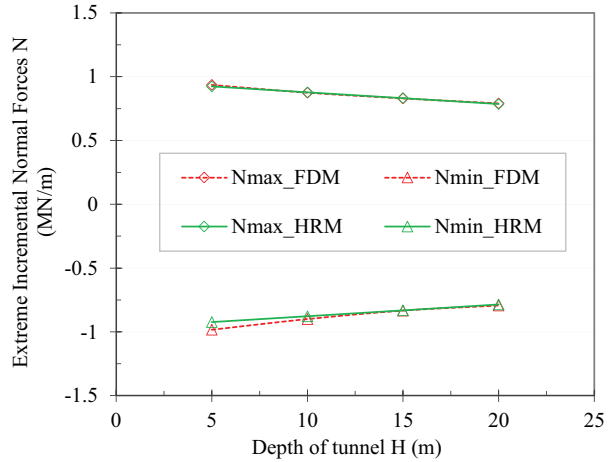
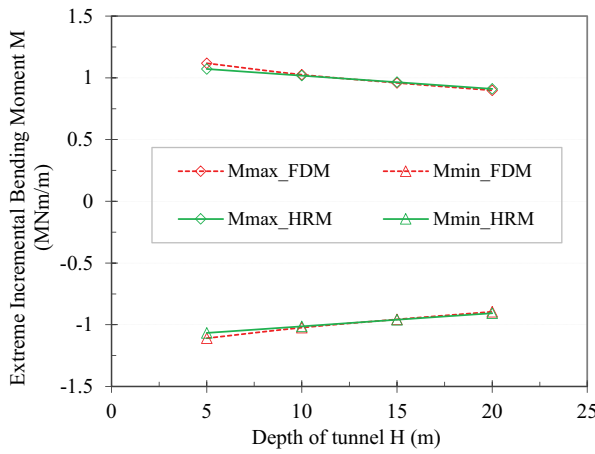
Figure 17 indicates that the extreme incremental bending moments decrease significantly by 24% when the tunnel burial depth changes from 5 to 20 m. Likewise, the extreme incremental normal forces



(a) Extreme incremental bending moments



(b) Extreme incremental normal forces

Figure 16. Effect of cross-section shape on the extreme incremental structural forces of the sub-rectangular tunnel lining.**Figure 17.** Effect of the tunnel burial depth on the extreme incremental structural forces of a sub-rectangular tunnel lining.

decrease gradually when the tunnel burial depth increases. A decrease of 18% of the extreme incremental normal forces obtained for a burial depth of 5 m compared to 20 m is observed. Figure 17 also indicates that the extreme incremental structural forces determined by the proposed HRM method are in good agreement with the numerical FDM computations. The discrepancies are, respectively, less than 6% and 4% corresponding to the extreme incremental normal forces and bending moments.

4.7. Validation 7

In this validation, the ground properties used by (Hashash, Park, and Yao (2005)) and (Sun, Du, and Dias (2021)) are adopted to demonstrate the accuracy of the proposed HRM method. The ground properties

are presented in Table 4. The other parameters are shown in Table 1 (i.e., $E_s = 100$ MPa, $t = 0.5$ m). The shear strains, γ_c , of 0.214%, 0.229%, and 0.297%, which correspond to the maximum horizontal acceleration, a_H , of 0.5 g are adopted for cases 1, 2, and 3, respectively. The comparisons between the HRM method and the numerical FDM ones are given in Table 5, in terms of extreme incremental bending moments and normal forces.

Table 5 shows a very good matching of the extreme incremental bending moments and normal forces developed in the tunnel lining calculated by the proposed HRM method and the numerical FDM for all three proposed cases. The differences are, respectively, smaller than 5.33% and 3.53% corresponding to the extreme incremental bending moments and normal forces.

Table 4. Ground properties (Hashash, Park, & Yao, 2005; Sun, Du, & Dias, 2021).

Case	Ground Parameter	Symbol	Unit	Value
1	Young's modulus	E_s	MPa	312
	Poisson's ratio	ν_s	-	0.3
2	Young's modulus	E_s	MPa	312
	Poisson's ratio	ν_s	-	0.49
3	Young's modulus	E_s	MPa	185.4
	Poisson's ratio	ν_s	-	0.49

Table 5. Comparison of the results of the HRM method and FDM model.

Case	Incremental structural forces	HRM method	FDM model	Difference (%)
Case 1	$M_{\max}$ (MNm/m)	0.723	0.748	-3.29
	$M_{\min}$ (MNm/m)	-0.719	-0.759	-5.33
	$N_{\max}$ (MN/m)	1.243	1.288	-3.53
	$N_{\min}$ (MN/m)	-1.242	-1.276	-2.68
Case 2	$M_{\max}$ (MNm/m)	0.748	0.746	0.34
	$M_{\min}$ (MNm/m)	-0.744	-0.746	-0.20
	$N_{\max}$ (MN/m)	1.174	1.162	1.00
	$N_{\min}$ (MN/m)	-1.173	-1.161	0.99
Case 3	$M_{\max}$ (MNm/m)	0.850	0.829	2.53
	$M_{\min}$ (MNm/m)	-0.846	-0.825	2.54
	$N_{\max}$ (MN/m)	0.955	0.950	0.54
	$N_{\min}$ (MN/m)	-0.954	-0.961	-0.70

5. Conclusions

The novelty and the scientific contribution of this study lie in proposing a new numerical procedure to efficiently calculate the behaviour of sub-rectangular tunnel linings subjected to seismic loading using the Hyperstatic Reaction Method. The HRM method under seismic conditions is presented, considering a large range of ground parameters, lining properties, tunnel dimensions and tunnel geometry. The entire computation procedure is assisted by parametric analysis and a pseudo-static loading scheme which was applied to the sub-rectangular tunnel lining is also proposed.

To verify the application capability of the developed HRM method, an extensive validation was performed considering a series of numerical computations. The developed HRM method was validated based on a comparison with a pseudo-static numerical FDM model. Comparative analyses revealed that the developed HRM method can be efficiently used in the preliminarily seismic design of sub-rectangular shaped tunnels.

The present study also indicates that in the case when a tunnel structure is more flexible than the groundmass, the tunnel lining will experience amplified distortions in comparison to the ground shear distortions in the free field. In contrast, when a tunnel lining is stiffer than the surrounding ground, it tends to resist ground displacements.

Although with the current numerical models' development and computing capacities, the proposed HRM method provides a new and alternative free method for

a fast and efficient seismic design of sub-rectangular tunnels. It is necessary to emphasize that the proposed method assumes an elastic ground-tunnel configuration subjected to the effect of ground shaking, ground and structure nonlinearities are therefore not covered by the presented method.

Highlights

- A new pseudo-static loading scheme applied to the sub-rectangular tunnel under seismic condition.
- Validation with finite difference models in different scenarios.
- The developed HRM method shows a good precision.

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Disclosure statement

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