

Optimal Design of Ground Geodetic - GNSS Networks Based on Baseline Configuration and Robustness Analysis

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Abstract: *The paper presents a study on the optimal design of ground geodetic networks using GNSS technology, focusing on baseline configuration and robustness analysis of the network under Vietnam's field conditions. The main objective is to propose solutions for network structuring and technical guidelines to enhance the stability and reliability of geodetic networks, minimize the impact of gross errors (outliers), and optimize surveying efficiency. The research methodology includes constructing a five-point network with measurement data consisting of 11 angles and 8 baselines, applying the indirect adjustment model, calculating redundancy numbers, and analyzing robustness indices under specific scenarios of baseline supplementation. Experimental results show that baselines II-IV and II-V play a key role, with very small redundancy numbers. When measured or supplemented, these baselines significantly improve the stability and reliability of the overall network adjustment. Robustness analysis further indicates that supplementing baseline II-IV helps reduce the maximum robustness indices at critical points (particularly points III and V) substantially, while also distributing reliability more evenly across the network points. These findings highlight the importance of optimizing network structures based on the theory of redundancy and robustness, and provide clear practical guidelines to enhance quality control performance in real-world GNSS surveys. The research conclusions support survey engineers and measurement organizations in selecting rational network structures, prioritizing the measurement or quality control of "backbone" baselines (with small redundancy numbers), in order to achieve both economic efficiency and high quality in ground geodetic projects.*

Keywords: GNSS, baseline, robustness, optimal design of geodetic network

1. Introduction

Ground geodetic networks play a pivotal role in providing an accurate spatial foundation for surveying, construction, and land management activities (Lu, Qu, & Qiao, 2014). Establishing a system of geodetic control points with precisely determined coordinates and elevations enables engineers and professionals to reliably identify the position, distance, and height of ground features (Schofield & Breach, 2007). In construction, geodetic networks are used to ensure the precise positioning of structures, guaranteeing that project components are executed according to design and technical specifications. In addition, they support the creation of detailed topographic maps and geographic information systems (GIS), serving urban planning, land management, boundary delineation, and sustainable urban development (Amiri-Simkooei, Asgari, Zangeneh-Nejad, & Zaminpardaz, 2012). Particularly in Vietnam, with its diverse terrain and complex environmental conditions, accurate geodetic networks are a decisive factor in reducing cumulative errors, enhancing efficiency, and ensuring the reliability of surveying and construction works (Tuan, 2019).

Global Navigation Satellite System (GNSS) technology is playing an increasingly important role in improving the accuracy and efficiency of ground geodetic networks (Lechner & Baumann, 2000). Compared with traditional surveying methods such as angle or distance measurements, GNSS enables more accurate positioning, is less dependent on terrain and weather conditions, and significantly reduces the time required for network establishment (Schaefer & Pearson, 2021). This technology does not require direct line-of-sight between observation points, making it especially convenient in areas with complex or inaccessible terrain. In addition, GNSS allows for the measurement of long distances while maintaining high accuracy, which facilitates the construction of wide-area control networks with appropriate point density (Deng, Lin, Phadke, Shukla, & Thorp, 2012). With continuous advancements and augmentation systems, GNSS provides high reliability and effectively meets the requirements of modern surveying, construction, and land management in both Vietnam and worldwide.

Optimal geodetic network design helps survey engineers maximize network efficiency. (Amiri-Simkooei et al., 2012) presented basic concepts of optimization and design of geodetic networks. The main goal is to ensure the network achieves the best possible quality, demonstrated through criteria such as precision, reliability, and cost. (Alizadeh Khameneh, 2017) presented the optimal design in geodetic GNSS-based networks. The results showed that an optimization problem can be analytically solved by classifying it into multiple orders, where the best configuration, datum, and observation weights can be found to meet the requirements for cost, precision, and dependability. In order to implement the optimal design of the GNSS network, various models were used. While (Hussein & Hasan, 2016) utilized the Neural Network Model, applied particle swarm optimization, and genetic algorithms for the second-order design of GNSS networks with different constraints, (Farhan, Gomaa, & Sedeek, 2022) used the Particle Swarm Optimization (PSO) algorithm to solve the geodetic Second-Order Design (SOD) problem for GNSS measurements. Its main goal is to select the optimal GNSS baselines to achieve the required accuracy at a reasonable cost, especially in cases with limited equipment (a small number of GNSS receivers). In addition, To solve the problem of optimizing the second-order design in geodesy, which is how to select the minimum number of measurements while still meeting the accuracy requirements, (Michal, 2017) proposed a method based on the gradual selection of the most beneficial measurements to meet the accuracy requirements. This method was successfully tested in optimizing leveling networks and has been shown to be remarkably effective when applied to GNSS networks, especially those using multiple. In another study, (Liu et al., 2021) presented the selection of optimal baselines in GNSS networks to improve positioning accuracy. The study applies a new weighting strategy combining baseline distance and the number of common observations to optimize the network structure. Experimental results using data from the global GNSS stations in 2012 show that the proposed method improves accuracy compared to traditional methods.

In Vietnam, research has also focused on the application of GNSS technology in establishing ground geodetic networks, particularly under diverse terrain and climatic conditions. Domestic studies emphasize the design of baseline configurations suited to local characteristics, combined with robustness analysis to minimize the influence of systematic errors and environmental factors such as rain, fog, and signal interference. Some studies have delved into developing adjustment methods and optimizing network structures to improve accuracy and stability in practical implementation. In addition, efforts have been made to establish quality assessment frameworks for GNSS networks to ensure the long-term sustainability and reliability of the geodetic system in Vietnam. (Tuan, 2019) has studied the method of optimal design and adjustment of mixed free-grid ground - GPS in Vietnam conditions, to process and analyze geodetic data containing coarse errors. The research results apply robust estimation method to improve the efficiency of geodetic data processing. However, there have been few discussions about the best technique for determining the independent baseline of a large GNSS network. Therefore, this study aims to optimize the design of ground geodetic networks using GNSS baseline configurations while analyzing the robustness of the network under practical conditions in Vietnam.

2. Algorithmic Model

2.1. Redundancy Number

In the indirect adjustment of geodetic networks, the system of error equations is established based on the least squares principle to optimize the correction of observations. The general form of the equations is (Phuc & Huong, 2016):

$$V = A\Delta X + L \quad (1)$$

Where V is the vector of observation corrections, A is the coefficient matrix relating the observations to the unknowns, ΔX is the vector of corrections to the unknowns, and L is the vector of raw or approximate observations (Phuc & Huong, 2016). From this, the system of normal equations is formulated.

$$R\Delta X + b \quad (2)$$

The unknown vector is defined as follows:

$$\Delta X = -(A^T P A)^{-1} A^T P L + L \quad (3)$$

$$\text{or} \quad \Delta X = (E - (A^T P A)^{-1} A^T P) L \quad (4)$$

$$\text{Put} \quad R_o = (E - A(A^T P A)^{-1} A^T P) \quad (5)$$

For the free network:

$$R_o = (E - A\tilde{R}A^T P) \quad (6)$$

At that time

$$\text{trace}(R_o) = \text{trace}(E) - \text{trace}(A\tilde{R}A^T P) \quad (7)$$

On the other hand

$$\sum r_{ii} = n - t = r \quad (8)$$

r_i is the residual value of the i -th observation

$$0 \leq r_i \leq 1, r = (0.5 - 0.7)t \quad (9)$$

Thus, the influence coefficient r_{ij} represents the extent to which the raw error affects the corresponding observation and is denoted as r_i . From Equation (9), it can be observed that the smaller the value of r_i , the greater the impact of the observation; conversely, the larger the value of r_i , the smaller the influence of the observation.

2.2. Robustness Algorithm

In geodetic network data processing, gross errors are abnormal observations that can significantly distort adjustment results if they are not detected and removed. The Robustness algorithm (also known as robust estimation) is a tool designed to enhance the “robustness” of parameter estimation, that is, to minimize the influence of observations containing gross errors on the final adjustment results (Berber, Dare, & Vaníček, 2006). Unlike the traditional Least Squares Estimation (LSE) method, which assumes that all observations are statistically correct (containing only random errors), robust estimation allows for the possibility that one or more outliers (gross errors) may exist in the dataset (Berber, 2023).

In principle, robust algorithms employ nonlinear weighting functions to “down-weight” or “reduce the influence” of observations suspected of containing gross errors. In addition to identifying problematic observations, these algorithms allow the assessment of solution robustness through robust indicators such as maximum residual, robust variance, and robust influence (Berber, Vaníček, & Dare, 2009). This enables the analyst to determine which observations should be removed, flagged, or reinforced with additional measurements. Consequently, robust algorithms enhance the reliability and practical value of geodetic coordinate networks, ensuring that large errors do not distort the model solution, thereby improving the quality, safety, and efficiency of modern surveying and mapping projects. According to (Baarda, 1968):

$$\Delta l_i = \sqrt{\lambda_0} \frac{\sigma_i}{\sqrt{r_i}} \quad (10)$$

Where:

- Δl_i is the smallest gross error in an observation detected by the statistical testing algorithm
- $\sqrt{\lambda_0}$ is the λ_{non} -centrality coefficient determined based on the test β_0 at a given α level
- σ_i is the standard deviation of the observation, and r_i is the residual of the i -th observation.

According to the least squares method, ΔX is determined as follows:

$$\Delta X = (A^T P A)^{-1} A^T P \Delta l \quad (11)$$

Where:

- A is the coefficient matrix of the adjustment equations for the observations
- P is the weight matrix of the observations, with dimensions $n \times n$. The correction vector ΔX can then be expressed as:

$$\Delta X = \begin{bmatrix} \Delta x_i \\ \Delta y_i \end{bmatrix} = \begin{bmatrix} u_i \\ v_i \end{bmatrix} \quad (12)$$

Where:

- u_i represents the measurement error along the X-axis
- v_i represents the measurement error along the Y-axis

Following the approaches presented in (Berber et al., 2006; Vaníček, Craymer, & Krakiwsky, 2001) the following relation holds for each point:

$$\mathbf{u}(T_j) = \mathbf{E}_i \cdot \mathbf{r}_{ij} + \mathbf{c} \quad (13)$$

Where:

- i is the number of points in the network $i = 1 \div t$
- j is the number of points connected to point i , $j = 1 \div k$, the coefficient matrix ($\mathbf{E}_i$) of point T_i has the form:

$$\mathbf{E}_i = \begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} \end{bmatrix} \quad (14)$$

In which, $\mathbf{r}_{ij}$ and $\mathbf{c}$ are calculated as follows:

$$\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i = \begin{bmatrix} x_{T_j} - x_{T_i} \\ y_{T_j} - y_{T_i} \end{bmatrix} \quad (15)$$

$$\mathbf{c} = \begin{bmatrix} u_x(T_i) \\ u_y(T_i) \end{bmatrix} = \begin{bmatrix} a_i \\ b_i \end{bmatrix} \quad (16)$$

Equation (4) can be expressed in the following form:

$$\mathbf{F}_{j,1} : \begin{bmatrix} 1 & (x_j - x_i) & (y_j - y_i) \end{bmatrix} \begin{bmatrix} a_i & \frac{\partial u_{xi}}{\partial x} & \frac{\partial u_{xi}}{\partial y} \end{bmatrix}^T = u_x(T_j) \quad (17)$$

$$\mathbf{F}_{j,2} : \begin{bmatrix} 1 & (x_j - x_i) & (y_j - y_i) \end{bmatrix} \begin{bmatrix} b_i & \frac{\partial u_{yi}}{\partial x} & \frac{\partial u_{yi}}{\partial y} \end{bmatrix}^T = u_y(T_j)$$

According to (Cai & Grafarend, 2007a, 2007b), formula (2) can be written as follows:

$$\begin{bmatrix} 1 & (x_1 - x_i) & (y_1 - y_i) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & (x_1 - x_i) & (y_1 - y_i) \\ 1 & (x_2 - x_i) & (y_2 - y_i) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & (x_2 - x_i) & (y_2 - y_i) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & (x_k - x_i) & (y_k - y_i) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & (x_k - x_i) & (y_k - y_i) \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} a_i \\ \frac{\partial u_{xi}}{\partial x} \\ \frac{\partial u_{xi}}{\partial y} \\ b_i \\ \frac{\partial u_{yi}}{\partial x} \\ \frac{\partial u_{yi}}{\partial y} \end{bmatrix} = \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ \vdots \\ u_{xk} \\ u_{yk} \\ u_{xi} \\ u_{yi} \end{bmatrix} \quad (18)$$

or

$$\mathbf{F} \cdot \text{vec}(\mathbf{E}_i^*) = \mathbf{u}_j \quad (19)$$

In which: $\mathbf{F}$ is the coefficient matrix, $\text{vec}(\mathbf{E}_i^*)$ is the coefficient vector to be calculated. Based on the least squares method, $\text{vec}(\mathbf{E}_i^*)$ is determined as follows:

$$\text{vec}(\mathbf{E}_i^*) = (\mathbf{F}^T \mathbf{P}_F \mathbf{F})^{-1} \mathbf{F}^T \mathbf{P}_F \cdot \mathbf{u}_j \quad (20)$$

P_F is the weight matrix, size is $k \times k$, S_{ij} is the distance between two points i and j .

$$P_{F_{ij}} = \frac{1}{1 + S_{ij}^2} \quad (21)$$

From $\text{vec}(\mathbf{E}_i^*)$ in formula (20), the characteristic values for each point of the network can be calculated as $\sigma_{\max}$, $\gamma_{\max}$, $\varpi_{\max}$

Where:

$$\sigma = \frac{e_{u_x} + e_{v_x}}{2} \quad (22)$$

$$\varpi = \frac{e_{u_x} - e_{v_x}}{2} \quad (23)$$

$$\gamma = \sqrt{\frac{1}{4}((e_{u_x} - e_{v_y})^2 + (e_{u_y} + e_{v_x})^2)} \quad (24)$$

3. Experimental Calculations

3.1 Network Scheme and Measurement Data

The ground control network consists of five points whose coordinates need to be determined. These points were arranged to ensure stability and optimize the adjustment process. The points are labeled I, II, III, IV, and V, and are marked with robust field monuments for convenient sighting and instrument setup. The measurement dataset includes (Table 1 and Table 2):

11 angles (observed between points to close the spatial network), and

8 baselines (distance measurements between selected pairs of points).

The baselines were chosen carefully to ensure that all points participate in the adjustment equations, avoiding “hanging points” and enhancing the ability to check measurements through the closure cycle. A typical network configuration may consist of a closed pentagonal polygon (five consecutive points forming a loop), combined with diagonal lines connecting non-adjacent points to increase redundancy and create a well-connected network. For baselines, if GNSS is applied, spatial vectors can be measured, or plane distances can be combined when using total stations, depending on the experimental setup. Fig. 1. Network diagram showing the layout of five control points (I–V) and the measured angles and baselines used for adjustment calculations.

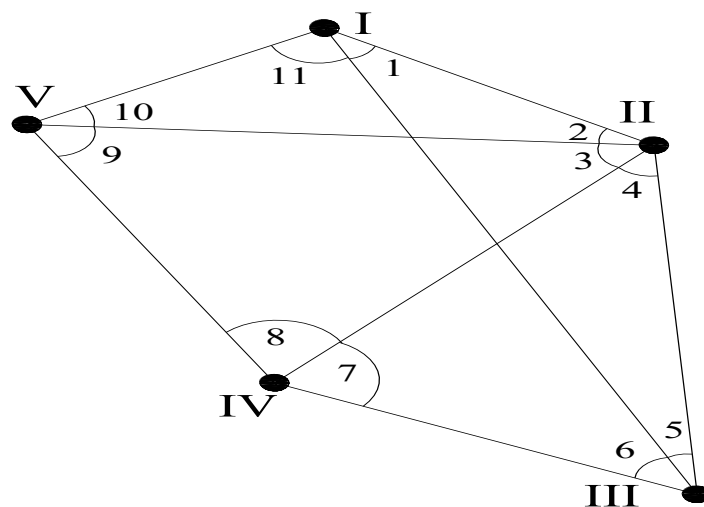


Fig. 1. Network diagram

Tab.1. Angle measurement data

No	Angle name			Angle value	
	Left	Interior	Right	Degree	Minute
1	II	I	III	33	27
2	V	II	I	28	34
3	IV	II	V	50	51
4	III	II	IV	46	35
5	I	III	II	20	30
6	IV	III	I	40	44
7	II	IV	III	72	8
8	V	IV	II	71	13
9	II	V	IV	57	55

Tab.2. Distance measurement data

No	Tên cạnh		S(m)
	Điểm đầu	Điểm cuối	
1	I	II	404.90
2	I	III	934.40
3	I	V	357.20
4	II	III	637.00
5	II	IV	586.80
6	II	V	655.70
7	III	IV	486.20
8	IV	V	537.15

The distance measurement error of the total station used in the experiment is characterized by the following formula

$$m_s = \pm(2\text{mm} + 1\text{ppm} \cdot D) \quad (25)$$

Where m_s is the standard error of the measured distance and D is the measured distance (m). **1 ppm** stands for **one part per million**, which represents the proportional error per unit of measured distance DDD . Specifically,

$$1\text{ ppm} \cdot D = (1/1,000,000) \cdot D \quad (26)$$

Thus, the distance measurement error of the total station is composed of a fixed component of 5 mm and a distance-dependent component of 1 ppm of the measured length, reflecting both the basic random error and the proportional error along the measured distance.

The angular measurement error of the total station (both horizontal and vertical angles) typically lies within:

$$m_\beta = \pm 3'' \quad (27)$$

where m_β denotes the standard accuracy of angle measurement for modern or medium-class instruments, as specified in technical standards. This value reflects the expected maximum deviation of observed angles during the experiment.

In addition, the combined distance and angular measurement errors are used as input parameters for constructing the weight matrix in the adjustment model. They also serve as the basis for assessing reliability, detecting, and eliminating abnormal observations in GNSS or conventional geodetic network analysis..

3.2 Results of the experimental calculations

3.2.1. Results of calculations based on redundancy

The experimental calculation results of the redundancy for each baseline are presented in Table 3. According to the results, the redundancy value r_i of each baseline clearly indicates the contribution of each observation to the overall reliability of the network:

The baselines I-V ($r = 0.979521$), IV-V ($r = 0.959297$), III-IV ($r = 0.958749$), II-III ($r = 0.957786$), and I-II ($r = 0.951983$) all have redundancy values greater than 0.95 (with r_i close to 1). This indicates that the influence of these baselines on the coordinate determination results is relatively limited, meaning that these observations can be removed without significantly affecting the overall network reliability. Therefore, in cases where measurement reduction is necessary (due to cost or time constraints), observations with high r_i values can be considered for reduction prior to those with very low r_i values.

In contrast, the baselines II-IV and II-V have the smallest redundancy values ($r_i = 2.5 \times 10^{-5}$ and $r_i = 4.2 \times 10^{-5}$, respectively), indicating that they have the greatest influence on the adjustment solution of the entire network. Additional measurements, quality control, or maintaining high accuracy for these baselines will significantly contribute to improving both the precision and robustness of the network.

$$\sum r_{ii} = 19, r = 0.7t = 7, 0.000028 \leq r_i \leq 0.986102$$

Tab 3. Ranking of baselines according to redundancy values

No.	Baselines	Value
1	I-V	0.979521
2	IV-V	0.959297
3	III-IV	0.958749
4	II-III	0.957786
5	I-II	0.951983
6	I-III	0.913492
7	II-V	0.000042
8	II-IV	0.000025

The above results demonstrate the role and practical significance of redundancy analysis in optimizing the network structure and selecting an efficient set of observations for GNSS networks. Designers have a solid scientific basis to selectively add or remove certain observations to balance economic efficiency with the reliability requirements of the project. Accurate determination of redundancy values also helps identify “critical” baselines for network stability (i.e., baselines with very small r_i) that should never be omitted or require careful supplementary measurements in practice. Specifically, in this study, it is recommended to prioritize additional measurements for baselines with small r_i values (e.g., II-IV, II-V) to enhance independence and reduce sensitivity to gross errors. At the same time, quality control measures should be applied to these observations, such as increasing the number of repetitions, checking measurement conditions, or using higher-precision instruments.

3.2.2. Experimental Calculations Using the Robustness Algorithm

In the experimental study, the Robustness algorithm was applied to evaluate the influence of individual observations on network stability. Robust indices (maximum residual, robust variance, robust influence) were computed for each control point under different baseline-addition scenarios, notably II-IV and II-V. The Robustness algorithm was applied to detect and quantify the influence of individual observations on network stability. Robust indices (the three indicators reported in the tables — hereafter

referred to as Indicator 1, Indicator 2 and Indicator 3) were computed for each control point under alternative baseline-supplement scenarios. Results are shown in Tables 4 and 5.

a. Comparison with non-critical baseline additions (Table 4).

Adding baselines IV–V or I–V results in relatively large robust-index values across the network: Indicator 1 averages about 19.7 when IV–V is added and 21.6 when I–V is added (values per point range ≈ 18.9 – 22.5). Indicator 2 and Indicator 3 are also larger for these scenarios (Indicator 2 averages ≈ 3.34 and 3.62 respectively). These high values indicate that supplementing with IV–V or I–V provides only modest improvement in global stability compared with the critical baselines discussed below (see Table 4).

b. Effect of adding critical baselines II–IV vs II–V

The scenarios that add baseline II–IV or II–V produce substantially lower robust indices, demonstrating clear improvement in network robustness (Table 5). Quantitatively:

- Indicator 1 (primary robust index):

- + Points I & II: $9.6 - 8.7$ (absolute reduction 0.9 , $\approx 9.4\%$).
- + Point III: $9.8 - 8.9$ (absolute reduction 0.9 , $\approx 9.2\%$).
- + Point IV: $9.6 - 8.5$ (absolute reduction 1.1 , $\approx 11.5\%$).
- + Point V: $8.7 - 7.6$ (absolute reduction 1.1 , $\approx 12.6\%$).

+ Average reduction of Indicator 1 across points $\approx 10.4\%$. These consistent decreases show that adding II–IV reduces the maximum residual-type measure across the network more effectively than adding II–V.

- Indicator 2 (secondary robust index):

- + Mostly unchanged at points I, II and IV ($1.3 - 1.3$).
- + Point III: $2.6 - 2.4$ ($\Delta = 0.2$, $\approx 7.7\%$).
- + Point V: $6.1 - 5.5$ ($\Delta = 0.6$, $\approx 9.8\%$).

+ Average reduction of Indicator 2 across points $\approx 3.5\%$. This shows modest but meaningful variance reductions at the more affected points (notably III and V).

- Indicator 3 (influence-type index):

- + Significant improvements occur at points most sensitive to gross errors:
- + Point III: $1.5 - 1.1$ ($\Delta = 0.4$, $\approx 26.7\%$).
- + Point V: $4.8 - 4.3$ ($\Delta = 0.5$, $\approx 10.4\%$).

+ Indicators at I, II, IV remain zero in both scenarios. These changes indicate that the influence of abnormal observations is reduced most markedly at points III and V when II–IV is added.

c. Relative improvement vs other additions

On average, supplementing with II–IV reduces Indicator 1 from ≈ 19.7 – 21.6 (the IV–V / I–V cases) down to ≈ 8.48 , i.e. a reduction of roughly 57–61% in the first robust index relative to those less effective baseline additions. Similar proportional improvements (≈ 30 – 35%) are observed for the secondary indices. This confirms that adding critical baselines (II–IV, II–V) yields much greater gains in global robustness than adding IV–V or I–V.

4. Practical interpretation and recommendation.

Both II–IV and II–V appear as critical baselines by redundancy analysis (very small r_i values in Table 3) and thus deserve priority in measurement and quality control. Robustness results indicate II–IV is preferred: it produces more uniform and larger reductions in the robust indices across control points, in particular delivering the largest relative improvements at the vulnerable points III and V. Operationally, the designer should therefore (a) prioritize surveying and repeated checks on II–IV (and II–V), (b) apply strengthened QC (additional repetitions, stricter field checks or higher-precision equipment) for these baselines, and (c) consider II–IV as the primary candidate when one baseline must be added to improve global stability.

From the experimental calculation results, it shows that when measuring additional baselines II–V, II–IV, the reliability of the grid increases significantly compared to the option of adding baselines: IV–V, I–V. Tables 4 and 5 demonstrate that targeted supplementation of network structure (notably II–IV) yields

measurable and substantial reductions in robustness metrics, confirming the value of combining redundancy-based selection with robustness analysis for optimal GNSS network design.

Tab.4. Comparison with non-critical baseline additions

Point	Robustness Algorithm					
	add baseline IV-V			add baseline I-V		
	$\sigma_{\max}$ (pmm)	$\gamma_{\max}$ (pmm)	$\varpi_{\max}$ (μrad)	$\sigma_{\max}$ (pmm)	$\gamma_{\max}$ (pmm)	$\varpi_{\max}$ (μrad)
I	19.7	2.1	0	21.6	2.4	0
II	19.7	2.1	0	21.6	2.4	0
III	20.5	3.5	2.4	22.5	3.7	2.7
IV	19.7	2.1	0	21.6	2.4	0
V	18.9	6.9	5.4	20.7	7.2	5.9

Tab. 5. Effect of adding critical baselines II–IV versus II–V

Point	Robustness Algorithm					
	add baseline II-IV			add baseline II-V		
	$\sigma_{\max}$ (pmm)	$\gamma_{\max}$ (pmm)	$\varpi_{\max}$ (μrad)	$\sigma_{\max}$ (pmm)	$\gamma_{\max}$ (pmm)	$\varpi_{\max}$ (μrad)
I	8.7	1.3	0	9.6	1.3	0
II	8.7	1.3	0	9.6	1.3	0
III	8.9	2.4	1.1	9.8	2.6	1.5
IV	8.5	1.3	0	9.6	1.3	0
V	7.6	5.5	4.3	8.7	6.1	4.8

5. Conclusion

Experimental calculations clearly show that the redundancy level of a single observation reflects its importance within the GNSS geodetic network. This redundancy concept plays a crucial role in the optimal design of ground and GNSS networks, as it provides designers with a scientific basis to decide which observations to add or remove. This is especially effective in the case of network renovation and improvement, leading to significant savings in time and cost during construction.

Furthermore, combining the optimal design method based on the redundancy of baselines with robustness analysis offers a powerful tool for designers to easily identify the best configuration. This approach not only improves the accuracy and stability of the network but also supports quality control measures to limit the impact of gross errors, ensuring sustainable and economically efficient survey operations.

The research results confirm that baselines with low redundancy values serve as the "backbone" of the network, necessitating priority in precise measurement and strict quality control to enhance the overall reliability of the geodetic network in practice.

Acknowledgment

The author thanks unnamed reviewers for their valuable comments which helped me to improve the quality of the manuscript.

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