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# ФИЗИКА

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**LINEAR APPROACH TO THE FLUTTER OF FGM CIRCULAR CYLINDRICAL SHELL FILLED INSIDE BY AN IDEAL UNCOMPRESSIBLE FLUID UNDER AERIAL MECHANICAL LOAD AND AERODYNAMICAL LOAD****Abstract**

This paper presents a linear analysis of flutter phenomena in functionally graded material (FGM) circular cylindrical shells filled with an ideal incompressible fluid, subjected to both aerial mechanical and aerodynamic loads. Two problems are formulated and solved: the quasi-static and dynamic linear stability analyses. The Galerkin method is employed to derive analytical expressions for determining the static and dynamic critical loads of the shell. Furthermore, the effects of geometric parameters and material property gradients on the critical loads and the structural response of the shells are investigated.

**Key words:**

flutter; functionally graded material; linear approach; static and dynamic critical loads; circular cylindrical shell.

**1. Introduction**

Functionally graded materials (FGMs) are a novel class of advanced composite materials that were first introduced in 1984 by a group of materials scientists in Japan, initially for the development of thermal barrier systems. These materials exhibit a gradual variation in composition and structure over their volume, resulting in corresponding changes in their mechanical properties from one surface to the other.

In recent years, FGMs have been increasingly utilized in high-temperature applications such as aerospace structures, fusion reactors, and other advanced engineering systems. Most FGMs consist of a combination of ceramic and metallic phases, allowing them to simultaneously leverage the beneficial properties of both materials—such as the excellent thermal and corrosion resistance of ceramics and the high strength, toughness, and bonding characteristics of metals.

Circular cylindrical shells made of FGMs are commonly employed in various engineering applications, including oil pipelines, aircraft fuel tanks, and rocket casings. Consequently, the vibration behavior and stability analysis of such structures have garnered significant attention from researchers worldwide.

The thermoelastic behavior of functionally graded cylindrical shells was analyzed by J.N. Reddy and C.D. Chin (1998). T. Prakash and M. Ganapathi (2006) investigated the supersonic flutter characteristics of FGM plates, including thermal effects. Nonlinear thermal flutter of FGM plates subjected to supersonic flow was presented by K.J. Sohn and J.H. Kim (2012). F. Sabri and A.A. Lakis (2013) employed the finite element method to analyze and predict the dynamic behavior of FGM cylindrical shells. The dynamic response of functionally graded cylindrical and conical shells traveling at supersonic speeds was discussed by H. Haddadpour, S. Mahmoudkhani, and H.M. Navazi (2008, 2010). Furthermore, Tran Quoc Quan, Dao Huy Bich, and Nguyen Dinh Duc (2014) investigated the nonlinear vibration of FGM cylindrical panels resting on an elastic foundation, using Ilyushin's (1956) supersonic aerodynamic theory.

**2. Basic formulations**

Consider a thin circular cylindrical shell as shown in Fig. 1, with mean radius  $R$ , length  $L$  and thickness subjected to axial compressive load  $\sigma_0$  and external airflow moving with velocity  $V$ . Cylindrical shell contains an ideal incompressible fluid. The middle surface of the shells is referred to the coordinates (...). The coordinate axis  $x$  is chosen in the generatrix direction of the shell, while the coordinate axis  $y$  and  $z$  respond

to the circumferential and thickness directions, respectively. The material properties of the FGM shell are varied continuously throughout its thickness direction. Young's modulus, density and Poisson's ratio are given by

$$E(z) = E_m V_m + E_c V_c = E_m + (E_c - E_m) \left( \frac{2z+h}{2h} \right)^k;$$

$$\rho(z) = \rho_m V_m + \rho_c V_c = \rho_m + (\rho_c - \rho_m) \left( \frac{2z+h}{2h} \right)^k; \nu(z) = \nu = \text{const}; k \geq 0. \quad (1)$$

According to Donnell's theory, the strain components across, curvature and torsion at middle surface are of the form

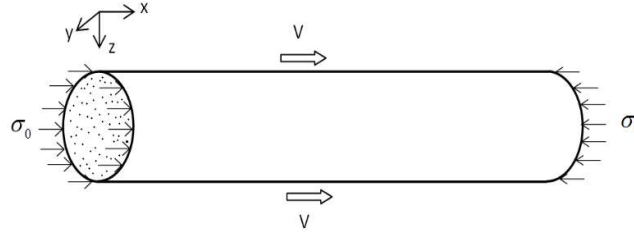


Figure 1 – Cylindrical Shell

$$\varepsilon_x^0 = \frac{\partial u}{\partial x}, \quad \varepsilon_y^0 = \frac{\partial v}{\partial y} - \frac{w}{R}, \quad \gamma_{xy}^0 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}; \quad \chi_x = \frac{\partial^2 w}{\partial x^2}, \quad \chi_y = \frac{\partial^2 w}{\partial y^2}, \quad \chi_{xy} = \frac{\partial^2 w}{\partial x \partial y}. \quad (2)$$

in which  $u, v, w$  are displacement components corresponding to the coordinates  $(x, y, z)$ ;

The geometrical compatibility equation is given as

$$\frac{\partial^2 \varepsilon_x^0}{\partial x^2} + \frac{\partial^2 \varepsilon_y^0}{\partial y^2} - \frac{\partial^2 \gamma_{xy}^0}{\partial x \partial y} = -\frac{1}{R} \frac{\partial^2 w}{\partial x^2} \quad (3)$$

The strain components across the shell thickness at a distance  $z$  from the middle surface are of the form

$$\varepsilon_x = \varepsilon_x^0 - z \chi_x, \quad \varepsilon_y = \varepsilon_y^0 - z \chi_y, \quad \gamma_{xy} = \gamma_{xy}^0 - 2z \chi_{xy}. \quad (4)$$

Hooke's law for cylindrical shells such as

$$\sigma_x = \frac{E(z)}{(1-\nu^2)} \left[ (\varepsilon_x^0 + \nu \varepsilon_y^0) - z (\chi_x + \nu \chi_y) \right],$$

$$\sigma_y = \frac{E(z)}{(1-\nu^2)} \left[ (\varepsilon_y^0 + \nu \varepsilon_x^0) - z (\chi_y + \nu \chi_x) \right], \quad \tau_{xy} = \frac{E(z)}{2(1+\nu)} (\gamma_{xy}^0 - 2z \chi_{xy}). \quad (5)$$

The stress-strain equations and their moments through the thickness of the shell are defined as

$$N_x = \frac{E_1}{(1-\nu^2)} (\varepsilon_x^0 + \nu \varepsilon_y^0) - \frac{E_2}{(1-\nu^2)} (\chi_x + \nu \chi_y),$$

$$N_y = \frac{E_1}{(1-\nu^2)} (\varepsilon_y^0 + \nu \varepsilon_x^0) - \frac{E_2}{(1-\nu^2)} (\chi_y + \nu \chi_x), \quad N_{xy} = \frac{E_1}{2(1+\nu)} \gamma_{xy}^0 - \frac{E_2}{1+\nu} \chi_{xy}. \quad (6)$$

$$M_x = \frac{E_2}{E_1} N_x - \frac{E_1 E_3 - E_2^2}{E_1 (1-\nu^2)} (\chi_x + \nu \chi_y), \quad M_y = \frac{E_2}{E_1} N_y - \frac{E_1 E_3 - E_2^2}{E_1 (1-\nu^2)} (\chi_y + \nu \chi_x), \quad (7)$$

$$M_{xy} = \frac{E_2}{E_1} N_{xy} - \frac{E_1 E_3 - E_2^2}{E_1 (1+\nu)} \chi_{xy}.$$

in which  $E_i$  is given by “H. Haddadpour (2008)”.

The equations of motion for the cylindrical shell, based on classical shell theory, are given by:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0, \quad (8)$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0, \quad (9)$$

$$\frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + \frac{N_y}{R} + q - \sigma_0 h \frac{\partial^2 w}{\partial x^2} = \rho_1 \frac{\partial^2 w}{\partial t^2} \quad (10)$$

### 3. Solution method

Put stress function  $F$  satisfies

$$N_x = \frac{\partial^2 F}{\partial y^2}, \quad N_y = \frac{\partial^2 F}{\partial x^2}, \quad N_{xy} = -\frac{\partial^2 F}{\partial x \partial y}. \quad (11)$$

Then the equation (8) and (9) correspond also equation (10) becomes

$$D \Delta \Delta w + \sigma_0 h \frac{\partial^2 w}{\partial x^2} - \frac{1}{R} \frac{\partial^2 F}{\partial x^2} - q + \rho_1 \frac{\partial^2 w}{\partial t^2} = 0, \quad (12)$$

In which  $D$  is given by “H. Haddadpour (2008)”.

Equation (12) contains two unknown functions  $w$  and  $F$ , and to find a second equation relating to these two functions, the geometrical compatibility equation (3) is used. For this aim, rather strain through stress and rather curvature, torsion through deflection, then substituting into the geometrical compatibility equation

$$\frac{1}{E_1} \Delta \Delta F = -\frac{1}{R} \frac{\partial^2 w}{\partial x^2} \quad (13)$$

Eqs. (12) and (13) are the governing equations used to investigate the flutter behavior of the system. In Eq. (12), the force term  $q$  accounts for aerodynamic force, aerodynamic damping, and the hydrodynamic force induced by the internal fluid, and is given by:

$$q = -\frac{\gamma}{g} h \varepsilon \frac{\partial w}{\partial t} + p + \frac{\gamma_L}{g} \frac{\partial \varphi_L}{\partial t}, \quad (14)$$

where  $\gamma$  is the specific weight of the air,  $g$  is the gravitational acceleration,  $\varepsilon$  is the aerodynamic damping coefficient,  $p$  is the aerodynamic pressure,  $\gamma_L$  is the specific weight of the fluid, and  $\varphi_L$  is the fluid velocity potential.

The aerodynamic pressure based on the linearized piston theory is expressed as:

$$p = -H p_\infty M \frac{\partial w}{\partial x}, \quad (15)$$

where  $p_\infty$  is the static pressure,  $H$  is the polytropic index,  $M = \frac{V}{V_\infty}$  is the Mach number,  $V$  is the flow velocity,  $V_\infty$  is the speed of sound. Substituting Eq. (14) into Eq. (12) yields:

$$D \Delta \Delta w + \sigma_0 h \frac{\partial^2 w}{\partial x^2} - \frac{1}{R} \frac{\partial^2 F}{\partial x^2} + \rho_1 \frac{\partial^2 w}{\partial t^2} + \frac{\gamma}{g} h \varepsilon \frac{\partial w}{\partial t} + H p_\infty M \frac{\partial w}{\partial x} - \frac{\gamma_L}{g} \frac{\partial \varphi_L}{\partial t} = 0 \quad (16)$$

The velocity potential of the fluid inside the cylindrical shell satisfies the governing equation written in cylindrical coordinates as:  $(x, \theta, r)$ ;  $0 < r < R$ ;  $0 < x < L$ ;  $0 < \theta < 2\pi$ .

$$\frac{\partial^2 \varphi_L}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi_L}{\partial r} + \frac{\partial^2 \varphi_L}{\partial x^2} + \frac{1}{r^2} \frac{\partial^2 \varphi_L}{\partial \theta^2} = 0. \quad (17)$$

subject to the boundary conditions:

$$\frac{\partial \varphi_L}{\partial r} = -\frac{\partial w}{\partial t} \text{ khi } r = R, \varphi_L \text{ must be finite at } r=0.$$

Assuming the shell is simply supported at the ends  $x = 0$  and  $x = L$  the shell's transverse displacement can be assumed in the form:

$$w = f_{mn}(t) \sin \frac{m\pi x}{L} \sin n\theta. \quad (18)$$

The velocity potential can be assumed in the separable form:

$$\varphi_L = A_{mn}(t) I_n \left( \frac{m\pi r}{L} \right) \sin \frac{m\pi x}{L} \sin n\theta, \quad (19)$$

where  $I_n$  is the radial part of the solution, typically represented by Bessel functions of the first kind  $f_{mn}(t)$ ,  $A_{mn}(t)$  is a time-dependent amplitude function to be determined. Applying the boundary conditions yields:

$$\varphi_L = -\frac{R I_n(\lambda_m)}{\lambda_m I_n'(\alpha R)} \frac{\partial w}{\partial t}; \lambda_m = \frac{m\pi R}{L} \quad (20)$$

Thus, the pressure exerted by the fluid on the shell wall, as a function of time, takes the form:

$$p_L = -\rho_L \frac{\partial \varphi_L}{\partial t} = \frac{\rho_L R I_n(\lambda_m)}{\lambda_m I_n'(\lambda_m)} \frac{\partial^2 w}{\partial t^2} = m_L \frac{\partial^2 w}{\partial t^2}. \quad (21)$$

#### 4. Assumed Mode Shape

To avoid trivial solutions when applying the Galerkin method to the cylindrical shell, the displacement function is assumed in a form that satisfies the boundary conditions:

$$w = f_{m,n}(t) \sin \alpha x \sin \beta y + f_{m+1,n+1}(t) \sin \tilde{\alpha} x \sin \beta y \quad (22)$$

where  $n$  is the circumferential wave number and  $m$  corresponds to half the axial wave number along the shell's length.

$$\alpha = \frac{m\pi}{L}; \beta = \tilde{\beta} = \frac{n}{R}; \tilde{\alpha} = \frac{(m+1)\pi}{L}; \theta = \frac{y}{R}.$$

#### 5. Quasi-Static Problem

Consider the case where the cylindrical shell is empty (i.e., contains no fluid), placed in an airstream, and subjected to a uniform compressive axial load of magnitude  $\sigma_0$ , but without the presence of vibration.

In this scenario, the inertial and damping terms  $\frac{\partial^2 w}{\partial t^2}$ ;  $\frac{\partial w}{\partial t}$ ;  $\frac{\partial \varphi_L}{\partial t}$  in Eq. (12) (or equivalently, Eq. (16)) are neglected. The resulting governing equations reduce to:

$$D \Delta \Delta w + \sigma_0 h \frac{\partial^2 w}{\partial x^2} - \frac{1}{R} \frac{\partial^2 F}{\partial x^2} + H p_\infty M \frac{\partial w}{\partial x} = 0 \quad (23)$$

$$\frac{1}{E_1} \Delta \Delta F = -\frac{1}{R} \frac{\partial^2 w}{\partial x^2} \quad (24)$$

Substituting the assumed mode shape from Eq. (22) into Eq. (24), the function  $F$  is obtained as:

$$F = \frac{E_1 \alpha^2}{R(\alpha^2 + \beta^2)^2} f_{m,n} \sin \alpha x \sin \beta y + \frac{E_1 \tilde{\alpha}^2}{R(\tilde{\alpha}^2 + \beta^2)^2} f_{m+1,n+1} \sin \tilde{\alpha} x \sin \beta y$$

Substituting this expression of  $F$  into Eq. (23) and applying the Galerkin method leads to the following system of equations:

$$\left( D(\alpha^2 + \beta^2)^2 - \sigma_0 h \alpha^2 + \frac{E_1 \alpha^4}{R^2 (\alpha^2 + \beta^2)^2} \right) f_{m,n} \frac{L\pi R}{2} - H p_\infty M \tilde{\alpha} f_{m+1,n+1} \frac{2mLR}{2m+1} = 0 \quad (25)$$

$$\left( D(\tilde{\alpha}^2 + \beta^2)^2 - \sigma_0 h \tilde{\alpha}^2 + \frac{E_1 \tilde{\alpha}^4}{R^2 (\tilde{\alpha}^2 + \beta^2)^2} \right) f_{m+1,n+1} \frac{L\pi R}{2} + H p_\infty M \alpha f_{m,n} \frac{2(m+1)LR}{2m+1} = 0 \quad (26)$$

For this linear system to have non-trivial solutions, the determinant of the coefficient matrix must vanish, yielding the characteristic equation:

$$\begin{aligned} \frac{L^2 \pi^2 R^2}{4} \left( D(\alpha^2 + \beta^2)^2 - \sigma_0 h \alpha^2 + \frac{E_1 \alpha^4}{R^2 (\alpha^2 + \beta^2)^2} \right) \left( D(\tilde{\alpha}^2 + \beta^2)^2 - \sigma_0 h \tilde{\alpha}^2 + \frac{E_1 \tilde{\alpha}^4}{R^2 (\tilde{\alpha}^2 + \beta^2)^2} \right) \\ + \frac{4m(m+1)L^2 R^2}{(2m+1)^2} H^2 p_\infty^2 M^2 \alpha \tilde{\alpha} = 0. \end{aligned} \quad (27)$$

This characteristic equation (27) is used to determine the critical load of the structure under quasi-static conditions.

### 6. Linear Dynamic Problem

Consider a cylindrical shell filled with an ideal incompressible fluid, placed in an airstream, and subjected to a uniform compressive axial load of magnitude  $\sigma_0$ . In this case, the equations governing the shell's dynamic behavior are:

$$D\Delta\Delta w + \sigma_0 h \frac{\partial^2 w}{\partial x^2} - \frac{1}{R} \frac{\partial^2 F}{\partial x^2} + H p_\infty M \frac{\partial w}{\partial x} + m_1 \frac{\partial^2 w}{\partial t^2} + \rho_k h \varepsilon \frac{\partial w}{\partial t} = 0, \quad (28)$$

$$\frac{1}{E_1} \Delta\Delta F = -\frac{1}{R} \frac{\partial^2 w}{\partial x^2}; m_1 = \rho_1 + m_L \quad (29)$$

Assuming the transverse displacement  $w$  takes the form given in Eq. (22), we obtain:

$$w = f_{m,n}(t) \sin \alpha x \sin \beta y + f_{m+1,n+1}(t) \sin \tilde{\alpha} x \sin \beta y \quad (30)$$

Following similar procedures and applying the Galerkin method, the system reduces to:

$$\begin{aligned} \left[ \left( D(\alpha^2 + \beta^2)^2 - \sigma_0 h \alpha^2 + \frac{E_1 \alpha^4}{R^2 (\alpha^2 + \beta^2)^2} \right) f_{m,n}(t) + m_1 f_{m,n}''(t) + \rho_k h \varepsilon f_{m,n}'(t) \right] \frac{L\pi R}{2} \\ - H p_\infty M \tilde{\alpha} f_{m+1,n+1}(t) \frac{2mLR}{2m+1} = 0 \end{aligned} \quad (31)$$

$$\begin{aligned} \left[ \left( D(\tilde{\alpha}^2 + \beta^2)^2 - \sigma_0 h \tilde{\alpha}^2 + \frac{E_1 \tilde{\alpha}^4}{R^2 (\tilde{\alpha}^2 + \beta^2)^2} \right) f_{m+1,n+1}(t) + m_1 f_{m+1,n+1}''(t) + \rho_k h \varepsilon f_{m+1,n+1}'(t) \right] \frac{L\pi R}{2} \\ + H p_\infty M \alpha f_{m,n}(t) \frac{2(m+1)LR}{2m+1} = 0 \end{aligned} \quad (32)$$

We seek solutions of the form  $f_{m,n}(t) = A_{m,n} e^{\lambda t}$ ;  $f_{m+1,n+1}(t) = A_{m+1,n+1} e^{\lambda t}$ . Substituting these into the system results in a pair of coupled linear algebraic equations for the coefficients  $A_{m,n}$  and  $A_{m+1,n+1}$ . For non-trivial solutions, the determinant of the coefficient matrix must vanish, leading to the characteristic equation:

$$\begin{aligned} B_1 B_2 + H^2 p_\infty^2 M^2 \tilde{\alpha} \alpha \frac{16m(m+1)}{(2m+1)^2 \pi^2} + (B_1 + B_2) m_1 \lambda^2 + (B_1 + B_2) \rho_k h \varepsilon \lambda \\ + m_1^2 \lambda^4 + \rho_k^2 h^2 \varepsilon^2 \lambda^2 + 2m_1 \rho_k h \varepsilon \lambda^3 = 0 \end{aligned} \quad (33)$$

$$B_1 = \left( D(\alpha^2 + \beta^2)^2 - \sigma_0 h \alpha^2 + \frac{E_1 \alpha^4}{R^2 (\alpha^2 + \beta^2)^2} \right); B_2 = \left( D(\tilde{\alpha}^2 + \beta^2)^2 - \sigma_0 h \tilde{\alpha}^2 + \frac{E_1 \tilde{\alpha}^4}{R^2 (\tilde{\alpha}^2 + \beta^2)^2} \right)$$

This equation is used to assess the stability of the shell under dynamic loading. The shell is considered stable if all real parts of the eigenvalues  $\lambda$  are negative. If any eigenvalue has a positive real part, the shell is dynamically unstable.

## 7. Numerical Results

### 7.1 Quasi-Static Problem

Consider a cylindrical shell with radius  $R=0.5$  m, and length  $L=2R$ , made from a functionally graded material composed of aluminum and alumina  $E_m = 70.10^9 \text{ N} / \text{m}^2$ ,  $\rho_m = 2702 \text{ kg} / \text{m}^3$  and  $E_c = 380.10^9 \text{ N} / \text{m}^2$ ,  $\rho_c = 3800 \text{ kg} / \text{m}^3$ . The Poisson's ratio is  $\nu = 0.3$ , the damping coefficient is taken as  $\varepsilon = 0.4$ , Mach number  $M=1.4$ , the static pressure  $\rho_\infty = 101400 \text{ N} / \text{m}^2$ , and the speed of sound  $V_\infty = 340 \text{ m} / \text{s}$ , and the shell moves within a steady airflow.

Tables 1a and 1b present the critical loads of the shell for varying geometric ratios and material gradient indices. As observed, increasing the  $R/h$  ratio leads to a decrease in the critical load. Similarly, as the material gradient index  $k$  increases, the shell's load-carrying capacity decreases. This result is expected, as a higher  $R/h$  corresponds to a thinner shell, and a higher  $k$  indicates a greater proportion of metal in the FGM, thus reducing the overall stiffness of the structure.

Table 2 shows that increasing the Mach number  $M$  results in a lower critical load. This is because higher flow velocities exert greater aerodynamic forces on the shell, making it more susceptible to instability. Table 3 illustrates the influence of modal indices  $(m,n)$  on the critical load. The minimum critical load is observed for the mode combination  $m=6, n=12$

Table 1a

Critical load for  $k=1$ , varying  $R/h$  Table 1b. Critical load for  $R/h=100$ , varying  $k$

| $R/h$ | $\sigma_0$  | $m$ | $n$ |
|-------|-------------|-----|-----|
| 100   | 225.955.500 | 10  | 11  |
| 150   | 150.591.100 | 11  | 14  |
| 200   | 112.953.300 | 5   | 13  |
| 250   | 90.353.000  | 15  | 18  |
| 300   | 75.295.110  | 15  | 20  |
| 500   | 45.190.110  | 33  | 1   |

| $k$ | $\sigma_0$ | $m$ | $n$ |
|-----|------------|-----|-----|
| 0   | 415949900  | 8   | 11  |
| 1   | 225955500  | 10  | 11  |
| 2   | 175210000  | 10  | 11  |
| 3   | 154180000  | 9   | 11  |
| 10  | 115695500  | 7   | 11  |

Table 2

Variation of the Critical Load with Respect to the Mach Number

| $n \backslash m$ | 1      | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 10     | 11     | 12     | 13     | 14     |
|------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1                | 3.4040 | 0.9503 | 0.2458 | 0.0959 | 0.0631 | 0.0731 | 0.0628 | 0.0673 | 0.0856 | 0.1168 | 0.1618 | 0.3007 | 0.3997 | 0.3997 |
| 2                | 1.8816 | 1.1557 | 0.5523 | 0.2414 | 0.1227 | 0.0766 | 0.0628 | 0.0650 | 0.0632 | 0.0714 | 0.0882 | 0.1134 | 0.1476 | 0.1916 |
| 3                | 1.1496 | 0.8520 | 0.5621 | 0.3347 | 0.1879 | 0.1147 | 0.0792 | 0.0647 | 0.0636 | 0.0627 | 0.0680 | 0.0796 | 0.0974 | 0.1214 |
| 4                | 0.7659 | 0.6270 | 0.4675 | 0.3303 | 0.2225 | 0.1447 | 0.1000 | 0.0758 | 0.0646 | x      | 0.0629 | 0.0675 | 0.0771 | 0.0913 |
| 5                | 0.5444 | 0.4723 | 0.3807 | 0.2921 | 0.2182 | 0.1598 | 0.1148 | 0.0870 | 0.0710 | 0.0637 | x      | 0.0634 | 0.0682 | 0.0769 |
| 6                | 0.4063 | 0.3657 | 0.3105 | 0.2526 | 0.2001 | 0.1566 | 0.1222 | 0.0947 | 0.0771 | 0.0670 | 0.0628 | 0.0625 | 0.0644 | 0.0697 |
| 7                | 0.3150 | 0.2906 | 0.2559 | 0.2173 | 0.1799 | 0.1471 | 0.1199 | x      | 0.0813 | 0.0703 | 0.0643 | x      | 0.0630 | 0.0661 |
| 8                | 0.2518 | 0.2364 | 0.2137 | 0.1874 | 0.1607 | 0.1359 | 0.1145 | 0.0970 | 0.0725 | 0.0659 | x      |        | 0.0643 | 0.0684 |
| 9                | 0.2065 | 0.1962 | 0.1809 | 0.1626 | 0.1433 | 0.1247 | 0.1080 | 0.9377 | 0.0823 | x      | 0.0671 | x      | 0.0634 | 0.0663 |
| 10               | 0.1730 | 0.1660 | 0.1553 | 0.1423 | 0.1626 | 0.1283 | 0.1143 | 0.1012 | 0.0899 | 0.0803 | 0.0729 | x      | x      | 0.0652 |

## 7.2 Linear Dynamic Problem

Consider a cylindrical shell with radius  $R=0.5$  m, and length  $L=2R$ , made from a functionally graded material composed of aluminum and alumina  $E_m = 70 \cdot 10^9 \text{ N/m}^2$ ,  $\rho_m = 2702 \text{ kg/m}^3$  and  $E_c = 380 \cdot 10^9 \text{ N/m}^2$ ,  $\rho_c = 3800 \text{ kg/m}^3$ . The Poisson's ratio is  $\nu = 0.3$ , the damping coefficient is taken as  $\varepsilon = 0.4$ , Mach number  $M=1.4$ , the static pressure  $p_\infty = 101400 \text{ N/m}^2$ , and the speed of sound  $V_\infty = 340 \text{ m/s}$ , and the shell moves within a steady airflow  $\rho_k = 1.293 \text{ kg/m}^3$ ,  $\rho_L = 1000 \text{ kg/m}^3$ . The shell is subjected to an axial compressive load  $\sigma_0 = 500000 \text{ Pa}$ .

Figures 1a–1d show the time histories of vibration amplitudes of the shell for  $L=6R$ ,  $R/h=200R$ , and  $k=1$ . At a flow velocity of 1285 m/s (Figure 1a), the system exhibits a beat-type oscillation with decaying amplitude. In Figure 1b, the amplitude gradually stabilizes at  $v=1285.7$  m/s, indicating a marginally stable state. However, in Figures 1c and 1d, where the velocities are  $v=1285.8$  m/s and  $v=1286$  m/s, respectively, the amplitude rapidly increases, signaling dynamic instability. Therefore, the critical flutter velocity can be approximated as  $v_{cr} \approx 1285.7$  m/s.

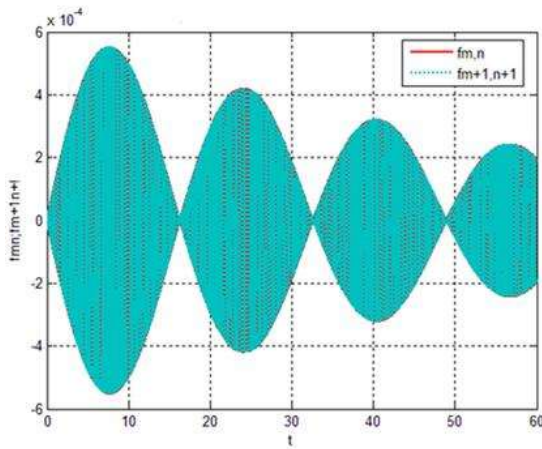


Figure 1a – Vibration amplitude at  $v=1285$  m/s

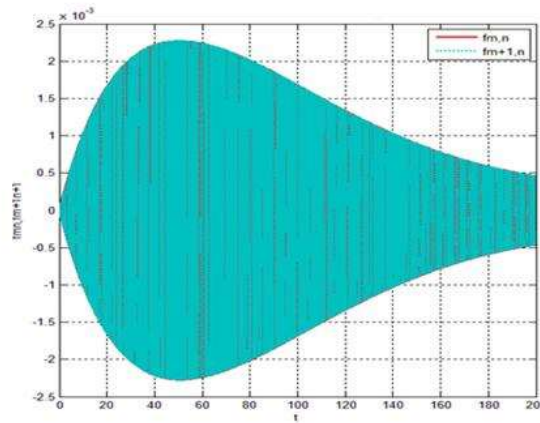


Figure 1b – Vibration amplitude at  $v=1285.7$  m/s

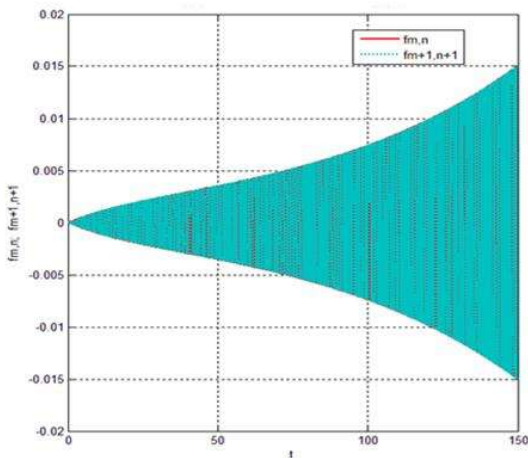


Figure 1c – Vibration amplitude at  $v=1285.8$  m/s

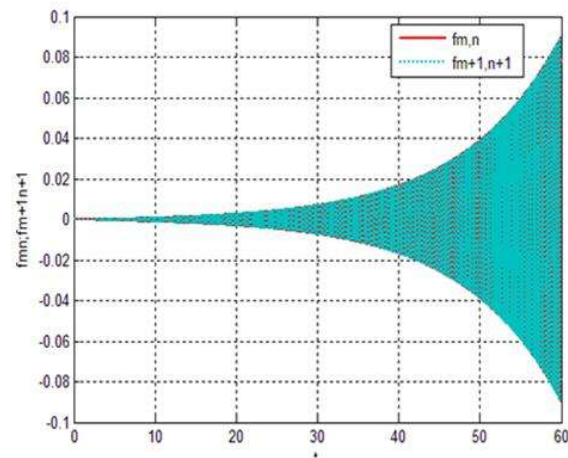


Figure 1d – Vibration amplitude at  $v=1286$  m/s

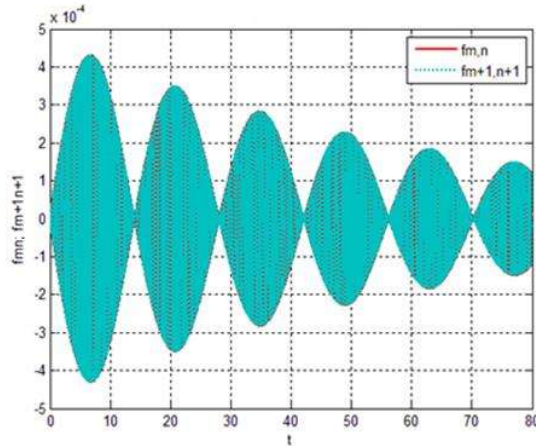


Figure 2a – Vibration amplitude at  $v=996$  m/s

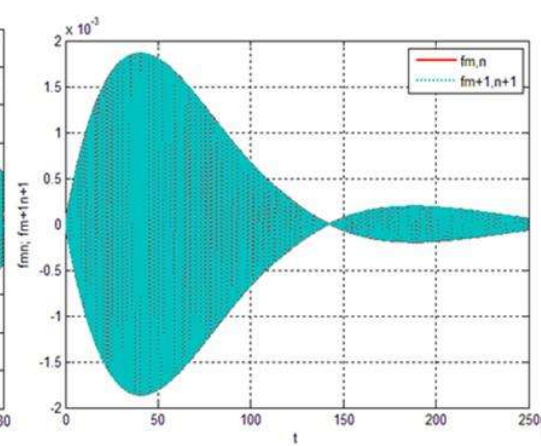


Figure 2b – Vibration amplitude at  $v=997.05$  m/s

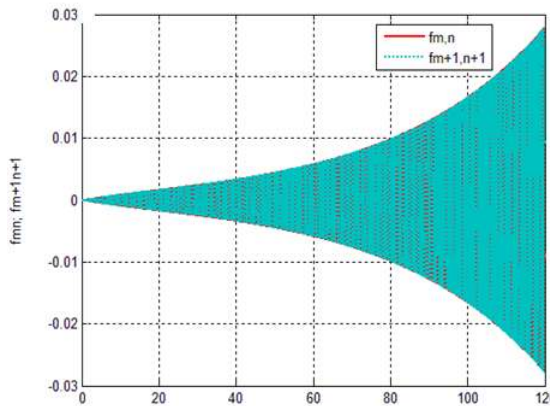


Figure 2c – Vibration amplitude at  $v=997.1$  m/s

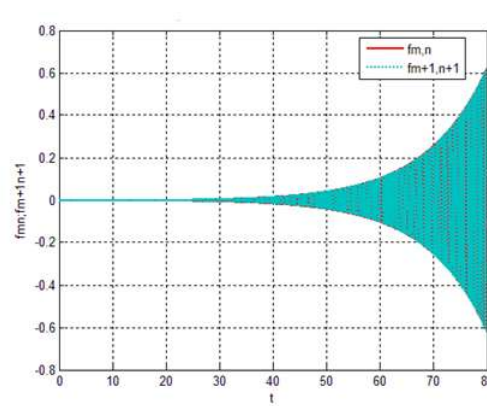


Figure 2d – Vibration amplitude at  $v=998$  m/s

Figures 2a–2d present the system's vibration response for  $k=2$  under the same geometrical conditions. At 996 m/s (Figure 2a), the amplitude shows a decaying beat pattern. At 997.05 m/s (Figure 2b), the response tends to stabilize. In contrast, at velocities of 997.1 m/s and 998 m/s (Figures 2c and 2d), the system becomes unstable as the amplitude grows rapidly. Thus, the critical flutter velocity in this case is approximately  $v_{cr} \approx 997.05$  m/

Table 4

Influence of  $R/h$  on critical velocity

| $R/h$ | Critical velocity with $L=6R$ | $m$ | $n$ |
|-------|-------------------------------|-----|-----|
| 100   | 6525.9                        | 1   | 7   |
| 200   | 1285.7                        | 1   | 9   |
| 300   | 495.4                         | 1   | 11  |
| 400   | 252.2                         | 1   | 12  |
| 500   | 149.3                         | 1   | 13  |

Table 5

Influence of  $L/R$  on critical velocity

| $L/R$ | Critical velocity with $R=500h$ | $m$ | $n$ |
|-------|---------------------------------|-----|-----|
| 1     | 3480.7                          | 1   | 18  |
| 2     | 932.6                           | 1   | 18  |
| 3     | 474.8                           | 1   | 16  |
| 4     | 294.8                           | 1   | 15  |
| 5     | 202.1                           | 1   | 14  |

Table 6

Influence of  $k$  on critical velocity

| $k$ | Critical velocity with<br>$R=6h, L=200h$ | $m$ | $n$ |
|-----|------------------------------------------|-----|-----|
| 0   | 2424.4                                   | 1   | 9   |
| 1   | 1285.7                                   | 1   | 9   |
| 2   | 997.1                                    | 1   | 9   |
| 3   | 883.9                                    | 1   | 9   |
| 4   | 824.4                                    | 1   | 9   |
| 5   | 786.2                                    | 1   | 9   |

Table 7

Influence of  $\sigma_0$  on critical velocity

| $s_0$   | Critical velocity with<br>$R=6h, L=200h$ | $m$ | $n$ |
|---------|------------------------------------------|-----|-----|
| 500000  | 1285                                     | 1   | 9   |
| 1000000 | 1281                                     | 1   | 9   |
| 1500000 | 1274                                     | 1   | 9   |
| 2000000 | 1269                                     | 1   | 9   |
| 3000000 | 1258                                     | 1   | 9   |
| 5000000 | 1235                                     | 1   | 9   |

Tables 4–7 illustrate the effects of geometric and material parameters on the critical flutter velocity. It is observed that increasing the  $L/R$  or  $R/h$  ratio, making the shell longer or thinner - results in a lower critical velocity. Furthermore, increasing the material gradient index  $k$ , which corresponds to a higher proportion of metal in the composition, reduces the shell stiffness and hence lowers the critical velocity. Similarly, an increase in the axial compressive force  $\sigma_0$  significantly decreases the critical flutter velocity.

### 8. Conclusion

This study presented a linear analytical approach to investigate the flutter behavior of functionally graded circular cylindrical shells filled with an ideal incompressible fluid under aerodynamic and mechanical loads. Both quasi-static and linear dynamic problems were formulated and solved. Using the Galerkin method, analytical expressions for static and dynamic critical loads were derived. Numerical simulations and time-domain responses were obtained using the Runge-Kutta method. The effects of geometric dimensions, material gradation, and flow velocity on the stability and dynamic response of the shell were thoroughly analyzed.

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