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FREQUENCY-DOMAIN VIBRATION THEORY OF FUNCTIONALLY GRADED MATERIAL BEAMS

Abstract

This paper presents the theoretical formulation for the vibration analysis of functionally graded material (FGM) beams in the frequency domain. Specifically, the study addresses the free vibration problem, including the determination of natural frequencies and corresponding mode shapes, as well as the formulations of the transfer (fundamental solution) matrix and the dynamic stiffness matrix. The proposed theory is illustrated through several numerical examples, including the evaluation of material constants, the determination of the neutral axis position, and the calculation of natural frequencies for representative FGM beam configurations.

Keywords:

Functionally graded material (FGM) beam; Free vibration; Natural frequency;
Dynamic stiffness matrix; Transfer matrix.

1. Equations of Motion

Consider an FGM beam of length (L) with a constant cross-section. The material properties are assumed to be uniform along the longitudinal and transverse directions, while varying continuously through the thickness ((z)-direction) according to the following power-law distribution:

$$\begin{Bmatrix} E(z) \\ G(z) \\ \rho(z) \end{Bmatrix} = \begin{Bmatrix} E_b \\ G_b \\ \rho_b \end{Bmatrix} + \begin{Bmatrix} E_t - E_b \\ G_t - G_b \\ \rho_t - \rho_b \end{Bmatrix} \left(\frac{z}{h} + \frac{1}{2} \right)^n, \quad -h/2 \leq z \leq h/2. \quad (1.1)$$

where (E), (G), and ρ denote Young's modulus, the shear modulus, and the mass density, respectively. The subscripts (t) and (b) refer to the material properties at the top and bottom surfaces of the beam. The coordinate (z) is measured from the mid-plane of the beam, whose thickness is (h). Under the assumption of small deformations, the displacement field is expressed as

$$u(x, t) = u_0(x, t) - (z - h_0)\theta(x, t); w(x, t) = w_0(x, t), \quad (1.2)$$

where $u_0(x, t)$ and $w_0(x, t)$ are the axial and transverse displacements of the neutral axis, respectively, (θ) denotes the rotation of the cross-section about the (y)-axis, and h_0 is the distance between the neutral axis and the mid-plane along the (z)-direction. The strain-displacement relationship is given by

$$\varepsilon_x = \partial u_0 / \partial x - (z - h_0) \partial \theta / \partial x; \gamma_{xz} = \partial w_0 / \partial x - \theta. \quad (1.3)$$

The corresponding constitutive relation is

$$\sigma_x = E(z)\varepsilon_x; \tau_{xz} = \psi G(z)\gamma_{xz}. \quad (1.4)$$

Assuming that the beam is subjected to a distributed transverse load $q(x, t)$, the equations of motion in the time domain can be derived using Hamilton's principle as

$$\begin{aligned} (I_{11}\ddot{u} - A_{11}u'') - (I_{12}\ddot{\theta} - A_{12}\theta'') &= 0; \\ (I_{12}\ddot{u} - A_{12}u'') - (I_{22}\ddot{\theta} - A_{22}\theta'') + A_{33}(w' - \theta) &= 0; \end{aligned} \quad (1.5)$$